Effect of quadratic density temperature variation on convective heat transfer flow of a viscous electrically conducting fluid through a porous medium in a non-uniformly heated vertical channel

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Abstract: we investigated the effects of quadratic density temperature variation on convection heat transfer flow of a viscous fluid through a porous medium in a non-uniform heated vertical channel in the presence of heat sources. The non linear equations governing the flow and heat transfer are solved by using the regular perturbation technique with the slope δ of the boundary temperature as the perturbation parameter. A velocity temperature and the rate of heat transfer are analyzed graphically for different variations of G, R, D^{-1}, M, α, α_1, and x.

Key Words: Heat transfer, Quadratic temperature, Porous medium

1.**Introduction**:

The magneto hydrodynamic heat transfer has gained significance in recent times owing to its applications in recent advancement of space technology. The MHD heat transfer can be divided arbitrarily into two sections. One contains problems in which the heat is an accidental byproduct of the electromagnetic fields (as in such MHD devices as generators, pumps etc.,) and the second consists of problems in which the primary use of electromagnetic fields is to control the heat transfer (as in the natural convection flows and aerodynamic heating). It is intuitively evident that the temperature distribution around a hot boundary in a fluid stream gives rise to a thermal boundary layer across which the temperature gradient is large. Hence the study of this thermal boundary layer and the influence of different forces on this layer is an important aspect of the heat transfer problems.

It is well known that in order to harness maximal geothermal energy one should have complete and precise knowledge of quanta of perturbation needed to initiate convection currents in mineral fluids embedded in the earth's crest enables one to use mineral energy to extract the minerals. For example, in the recovery of hydrocarbons from underground petroleum reservoirs, the use of thermal processes is becoming important to enhance the recovery. Heat can be injected into the reservoir as hot water or stream or heat can be generated institute by burning part of the reservoir crude. In all such thermal recovery processes fluid flow takes place through a porous medium and convection flow through a porous medium if of almost important, determination of the external energy required to initiate convection currents needs a through understanding of convective processes in a porous medium. There has been a great quest in Geophysicists to study the problem of convection currents in a porous medium heated from below.

In recent years, the convective transport mechanisms arising from these diurnal heating and cooling cycles have attracted significant attention from researchers, and many reports on topics of horizontal exchange flows can be found in the literature[18-23].However, previous investigations are limited in general to simple geometries such as a triangular cavity, which is a poor representation of real systems, although the triangular cavity is a good starting point for further investigations.

A liquid coolant is widely used to prevent the overheating of heat transfer rate improvement of equipments such as electronic devices, heat exchangers and transportation vehicles. However, conventional heat transfer fluid such as water or ethylene glycol generally has poor thermal properties. So, many efforts for dispersing small particles with high thermal conductivity in the liquid coolant have been conducted to enhance thermal properties of the conventional heat transfer fluids. This motivations leads to development of nanofluids [1-23].

In fact, Barletta et al[15] discussed viscous dissipation affects strongly the heat transfer process whenever the operating fluid has a low thermal conductivity, a high viscosity and flows in ducts with a small cross-section and a small wall heat flux.All these features may occur,for instance, in the micro channel flows considered for the design of MEMS.

More recently, studies of the viscous dissipation effect in laminar duct flows have been performed in order to include the ases of slug velocity profile, of slip flow in microtubes and of Non-Newtonian fluid behaviour [1-4]. In view of this several authors notably, Soudalgekar and Pop[14], Barletta [3,4], El-hakeing [8], Bulent Yesilata[7], Rossidi schio[5], Israel et al [9] Raveendra nath et al[16],Barletta et al [20] have studied the effect of viscous dissipation on the convective flows past on infinite vertical plates and through vertical channels and Ducts. The effect of viscous dissipation on natural convection has been studied for some different cases including the natural convection from horizontal cylinder.

2.**Formulation of the problem**

We analyze the steady motion of viscous, incompressible fluid through a porous medium in a vertical channel bounded by flat walls which are maintained at a non-uniform wall temperature in the presence of a constant heat source . A uniform magnetic field of strength is applied normal to the walls . The Boussinesq approximation is used so that the density variation will be considered only in the buoyancy force. The viscous dissipations and the joule heating are taken into account in the energy equation. Also the kinematic viscosity ν, the thermal conducting k are treated as constants. We choose a rectangular Cartesian system 0(x ,y) with x-axis in the vertical direction and y-axis normal to the walls.The walls of the channel are at y = ± L. The equations governing the steady flow , heat and mass transfer are

Equation of continuity:

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \qquad (2.1)$$

Equation of linear momentum**:**

$$\rho_e\left(u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}\right)=-\frac{\partial p}{\partial x}+\mu\left(\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}\right)-\rho g-(\sigma\mu_e^2 H_o^2)u-\left(\frac{\mu}{k}\right)u \qquad (2.2)$$

$$\rho_e\left(u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y}\right)=-\frac{\partial p}{\partial y}+\mu\left(\frac{\partial^2 v}{\partial x^2}+\frac{\partial^2 v}{\partial y^2}\right)\left(\frac{\mu}{k}\right)u \qquad (2.3)$$

Equation of Energy:

$$\rho_e C_p\left(u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}\right)=\lambda\left(\frac{\partial^2 T}{\partial x^2}+\frac{\partial^2 T}{\partial y^2}\right)+Q+(T_e-T) \qquad (2.4)$$

Equation of State:

$$\rho-\rho_e=-\beta\rho_e(T-T_e)^2 \qquad (2.5)$$

where ρ_e is the density of the fluid in the equilibrium state, Te is the temperature in the equilibrium state,(u,v) are the velocity components along O(x ,y) directions, p is the pressure, T is the temperature in the flow region, ρ is the density of the fluid,μ is the constant coefficient of viscosity , Cp is the specific heat at constant pressure,λ is the coefficient of thermal conductivity , μ_e is the *the magnetic* permeability, σ is the electrical conductivity , β is the coefficient of thermal expansion, Q is the strength of the internal heat source. k is the permeability of the porous medium.
In the equilibrium state

$$0 = -\frac{\partial p_e}{\partial x} - \rho_e g \tag{2.6}$$

Where $p = p_e + p_D, p_D$ being the hydrodynamic pressure.
The flow is maintained by a constant volume flux for which a characteristic velocity is defined as

$$Q = \frac{1}{2L}\int_{-L}^{L} u\, dy. \tag{2.7}$$

The boundary conditions for the velocity and temperature fields are

$$u = 0 \;,\; v = 0 \qquad \text{on } y = \pm L$$
$$T - T_e = \gamma(\delta x / L) \quad \text{on } y = \pm L \tag{2.8}$$

γ is chosen to be twice differentiable function,δ is a small parameter characterizing the slope of the temperature variation on the boundary.
In view of the continuity equation we define the stream function ψ as

$$u = -\psi_y\,,\; v = \psi_x \tag{2.9}$$

the equation governing the flow in terms of ψ are

$$\left[\frac{\partial\psi}{\partial x}\frac{\partial(\nabla^2\psi)}{\partial y} - \frac{\partial\psi}{\partial y}\frac{\partial(\nabla^2\psi)}{\partial x}\right] = \nu\left(\frac{\partial^2\psi}{\partial x^2} + \frac{\partial^2\psi}{\partial y^2}\right)^2 - 2(T - T_e)\beta g\frac{\partial T}{\partial y} - \left(\frac{\sigma\mu_e^2 H_o^2}{\rho}\right)\frac{\partial^2\psi}{\partial y^2} - \left(\frac{\mu}{k}\right)\nabla^2\psi \tag{2.10}$$

$$\rho_e C_p\left(\frac{\partial\psi}{\partial y}\frac{\partial\theta}{\partial x} - \frac{\partial\psi}{\partial x}\frac{\partial\theta}{\partial y}\right) = \lambda\nabla^2\theta + Q + (T - T_e) \tag{2.11}$$

Introducing the non-dimensional variables in (2 .10) - (2.11) as

$$(x',\ y') = (x, y)/L,\ (u', v') = (u, v)/U, \theta = \frac{T - T_e}{\Delta T_e}$$

$$p' = \frac{p_D}{\rho_e U^2} \quad ,\gamma' = \frac{\gamma}{\Delta T_e} \tag{2.12}$$

(under the equilibrium state $\Delta T_e = T_e(L) - T_e(-L) = \frac{QL^2}{\lambda}$)

the governing equations in the non-dimensional form (after dropping the dashes) are

$$R\frac{\partial(\psi, \nabla^2\psi)}{\partial(x, y)} = \nabla^4\psi + 2\frac{G\theta}{R}\theta_y - M^2\frac{\partial^2\psi}{\partial y^2} - D^{-1}\nabla^2\psi \tag{2.13}$$

and the energy diffusion equations in the non-dimensional form are

$$P_1 R\left(\frac{\partial\psi}{\partial y}\frac{\partial\theta}{\partial x} - \frac{\partial\psi}{\partial x}\frac{\partial\theta}{\partial y}\right) = \nabla^2\theta - \alpha\theta \tag{2.14}$$

where

$R = \frac{UL}{\nu}$ (Reynolds number)

$G = \frac{\beta g \Delta T_e L^3}{\nu^2}$ (Grashof number)

$M^2 = \frac{\sigma\mu_e^2 H_o^2 L^2}{\upsilon^2}$ (HartmannNumber)

$D^{-1} = \frac{L^2}{k}$ (Darcy Parameter)

$\mathrm{P} = \frac{\mu c_p}{k_1}$ (Prandtl number)

$\alpha = \frac{QL^2}{C_p K_f}$ (Heat Source Parameter)

The corresponding boundary conditions are

$$\psi(+1) - \psi(-1) = 1$$

$$\frac{\partial\psi}{\partial x} = 0, \quad \frac{\partial\psi}{\partial y} = 0 \quad at\ y = \pm 1 \tag{2.15}$$

$$\theta(x, y) = f(\delta x) \quad on\ \ y = \pm 1 \tag{2.16}$$

$$\frac{\partial\theta}{\partial y} = 0 \ \ at \quad y = 0 \tag{2.17}$$

The value of ψ on the boundary assumes the constant volumetric flow in consistent with the hypothesis(2.7) .Also the wall temperature varies in the axial direction in accordance with the prescribed arbitrary function γ(x) .

3.Rate of Heat transfer

The rate of heat transfer at the walls y=±1 in the non dimensional form is given by

$$\mathrm{Nu} = \frac{1}{\theta_m - \gamma(\frac{\partial\theta}{\partial y})} \quad \text{at } y=\pm 1$$

and corresponding expressions are,

$$(\mathrm{Nu})_{y=+1} = \frac{a_{49} + \delta a_{51}}{\theta_m - \gamma(x)} \quad (\mathrm{Nu})_{y=-1} = \frac{a_{50} + \delta a_{52}}{\theta_m - \gamma(x)}$$

where a_{49}, a_{50} and a_{54} are constants

4.**Discussion of the numerical results:**

In this analysis we investigate the effect of quadratic density temperature variation on convective heat transfer flow of a viscous electrically conducting fluid to a porous medium in a non uniformly heated vertical channel in the presence of heat generating sources. The non linear equations governing the flow and heat transfer are solved by employing a regular perturbation parameter. In this analysis we take a Prandtl number P=0.71 and δ=0.71.

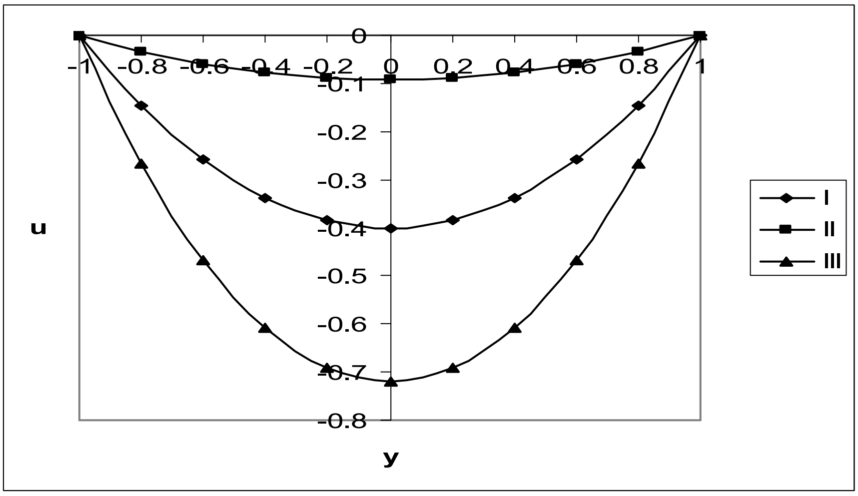

Fig. 1 : Variation of u with α

	I	II	III
α	2	4	6

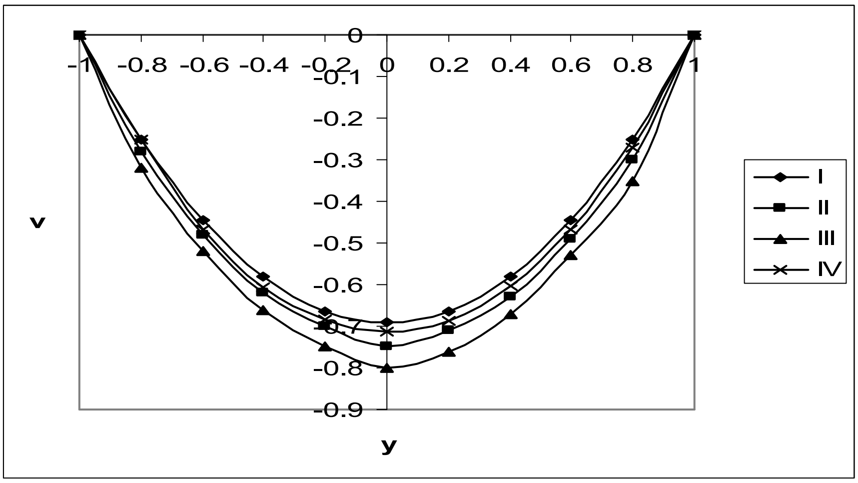

Fig. 2 : Variation of u with α_1

	I	II	III	IV
α_1	0.1	0.3	0.4	0.5

A variation of u with heat source parameter α shows that |u| depreciates with increase in $\alpha \leq 4$ and enhances with higher $x \geq 6$ (Fig.1). The effect of non uniform boundary temperature on u is shown in Fig-1. It is found that an increase in amplitude α_1 of the boundary temperature results in an enhancement in |u| and depreciates with higher $\alpha_1 \geq 0.5$ (Fig.2).

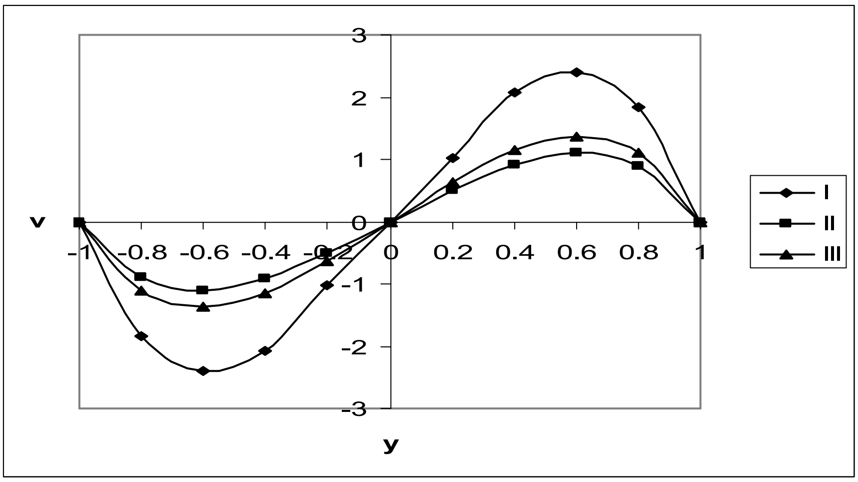

Fig. 3 : Variation of v with α

	I	II	III
α	2	4	6

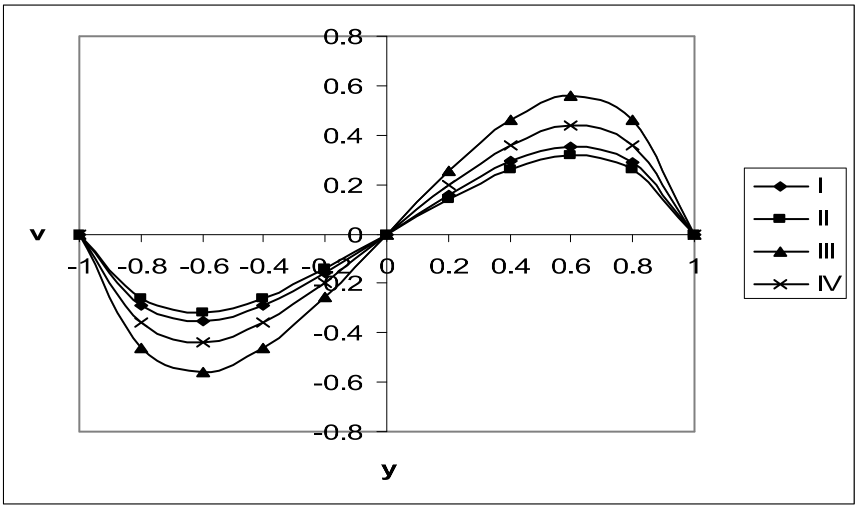

Fig. 4 : Variation of v with α_1

	I	II	III	IV
α_1	0.1	0.3	0.4	0.5

An increase in $\alpha \leq 4$ depreciates |v| and for higher $\alpha \geq 6$ we notice an enhancement in |v| (Fig.3). The effect of non uniform boundary temperature on v is shown in Fig.4 It is found that |v| depreciates with increase in $\alpha_1 \leq 0.3$, enhances at α_1=0.4 and again depreciates with $\alpha_1 \geq 0.5$.

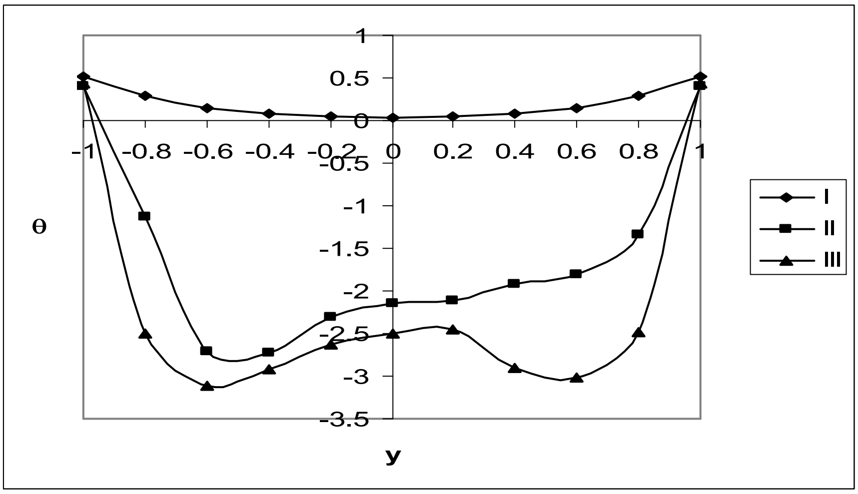

Fig. 5 : Variation of v with α

	I	II	III
α	2	4	6

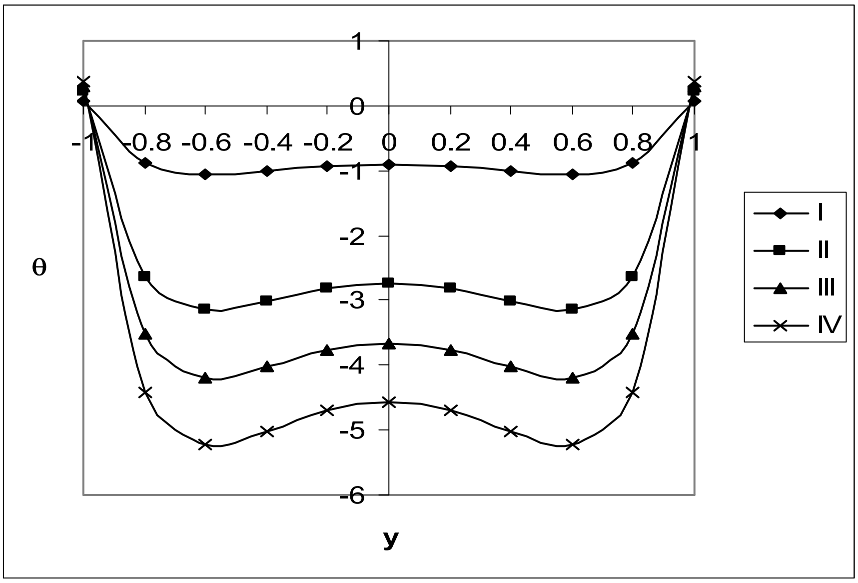

Fig. 6 : Variation of v with α_1

	I	II	III	IV
α_1	0.1	0.3	0.4	0.5

A variation of θ with heat source parameter α shows that the actual temperature experiences a depreciation with α in the entire flow region (Fig.5). An increase in the amplitude α_1 of the boundary temperature reduces the actual temperature in the flow region (Fig.6).

Table. 1_Average Nusselt Number (Nu) at y = +1

G	I	II	III	IV	V
$1x10^3$	-0.56524	-0.55760	-0.55275	-0.56354	-0.56200
$3x10^3$	-0.56505	-0.55745	-0.55263	-0.56406	-0.56215
$-1x10^3$	-0.56544	-0.55775	-0.55287	-0.56301	-0.56184
$-3x10^3$	-0.56563	-0.55790	-0.55299	-0.56249	-0.56169
R	35	70	140	35	35
D^{-1}	$1x10^2$	$1x10^2$	$1x10^2$	$3x10^2$	$5x10^2$

Table 2_Average Nusselt Number (Nu) at y = +1

G	I	II	III	IV	V
$1x10^3$	-0.56524	-0.55987	-0.54189	-0.57383	-0.60019
$3x10^3$	-0.56505	-0.56000	-0.54204	-0.57381	-0.60019
$-1x10^3$	-0.56544	-0.55974	-0.54175	-0.57384	-0.60019
$-3x10^3$	-0.56563	-0.55961	-0.54161	-0.57386	-0.60020
M	2	4	10	2	2
α	2	2	2	4	6

Table 3 Average Nusselt Number (Nu) at y = +1

G	I	II	III	IV	V	VI	VII
$1x10^3$	-0.56532	-0.56528	-0.56524	-0.56521	4.40706	-0.53446	-0.53790
$3x10^3$	-0.56528	-0.56516	-0.56505	-0.56494	4.40709	-0.53448	-0.53785
$-1x10^3$	-0.56536	-0.56540	-0.56544	-0.56547	4.40704	-0.53443	-0.53796
$-3x10^3$	-0.56540	-0.56552	-0.56563	-0.56574	4.40701	-0.53440	-0.53801
α_1	0.1	0.3	0.5	0.7	0.5	0.5	0.5
X	$\frac{\pi}{4}$	$\frac{\pi}{4}$	$\frac{\pi}{4}$	$\frac{\pi}{4}$	$\frac{\pi}{2}$	π	2π

Table 4_Average Nusselt Number (Nu) at y = -1

G	I	II	III	IV	V
$1x10^3$	0.56545	0.55776	0.55288	0.56295	0.56183
$3x10^3$	0.56567	0.55794	0.55302	0.56231	0.56165
$-1x10^3$	0.56523	0.55759	0.55274	0.56360	0.56201
$-3x10^3$	0.56501	0.55742	0.55260	0.56424	0.56219
R	35	70	140	35	35
D^{-1}	$1x10^2$	$1x10^2$	$1x10^2$	$3x10^2$	$5x10^2$

Table 5 Average Nusselt Number (Nu) at y = -1

G	I	II	III	IV	V
$1x10^3$	0.56545	0.55973	0.54174	0.57385	0.60019
$3x10^3$	0.56567	0.55957	0.54158	0.57387	0.60020
$-1x10^3$	0.56523	0.55988	0.54194	0.57383	0.60019
$-3x10^3$	0.56501	0.56003	0.54207	0.57381	0.60019
M	2	4	10	2	2
α	2	2	2	4	6

Table.6 _Average Nusselt Number (Nu) at y = -1

G	I	II	III	IV	V	VI	VII
$1x10^3$	0.56536	0.56541	0.56545	0.56550	-4.40705	0.53443	0.53796
$3x10^3$	0.56541	0.56554	0.56567	0.56581	-4.40704	0.53439	0.53802
$-1x10^3$	0.56532	0.56527	0.56523	0.56518	-4.40705	0.53446	0.53790
$-3x10^3$	0.56527	0.56514	0.56501	0.56487	-4.40706	0.53449	0.53784
α_1	0.1	0.3	0.5	0.7	0.5	0.5	0.5
X	$\frac{\pi}{4}$	$\frac{\pi}{4}$	$\frac{\pi}{4}$	$\frac{\pi}{4}$	$\frac{\pi}{2}$	π	2π

The average Nusselt number (Nu) which measures the local rate of heat transfer at the boundaries y=±1 is shown in tables 1-6 for different values of G, R, D^{-1}, M, α, α_1 and x. It is found that the rate of heat transfer depreciates at y=±1 and increases at y=-1with increase in G > 0, while for an increase in G < 0, |Nu| enhances at y=+1 and depreciates at y= -1. An increase in R or D^{-1} depreciates |Nu| at both the walls (Tables 1&4). The variation of Nu with Hartman number N shows that higher the Lorenge force smaller |Nu| at y=±1. An increase in the strength of heat source results in an enhancement in |Nu| at y=±1 (Tables 2&5). The variation of Nu with amplitude α_1 of the boundary temperature shows that higher the amplitude α_1 smaller |Nu| at y = +1 and larger |Nu| at y = -1. Moving along the axial direction of the channel walls the rate of heat transfer depreciates with x≤π/2 and enhances with higher x≥π (Tables 3&6).

5.**References**:

[1] A.Barletta,E.Zanchini,Forced Convection in the thermal entrance region of a circular duct with slug flow and viscous dissipation**,**Int.J.Heat mass transfer.*40(1997)1181-1190*

[2] A.Barletta,Slug flow heat transfer in circular ducts with viscous dissipation and convective boundary conditions, Int.J.Heat mass transfer 40(1997) 4219-4228.

[3] G.Tunc,Y.Bayazitoglu,Heat transfer in microtubes with viscous dissipation**,** Int.J.Heat mass transfer44(2001)2395-2403.

[4] P*.J.Oliveira,P.M.Coelho,F.T.Pinho,*The Graetz problem with viscous dissipation for FENE-P fluids,J.Non-Newtonian fluid Mech.121(2004)69-72.

[5] A.Barletta, E.Magyari, Forced convection with viscous dissipation in the thermal entrance region of a circular duct with prescribed wall heat flux,Int.J Heat mass transfer 50 (2007)26-35.

[6] G.Ramanaiah, G. Malarvizhi, Free convection on a horizontal plate in a saturated porous medium with prescribed heat transfer coefficient, Acta. Mech. 87(1991)73-80.

[7] **A.**A.Raptis, Unsteady free convective flow through a porous medium, Int. J. Engg. Sci,

21,(1983)345-348.

[8] K.Vajravelu, K.S.Sastri, Fully developed laminar free convection flow between two parallel vertical walls-1. Int.J Heat mass transfer 20 (1977)655-660.

[9] A.Barletta, Fully developed laminar forced convection in circular ducts for power-law fluidswith viscous dissipation, Int.Heat mass transfer.*40(1997)15-26.*

[10] V.M.Soundalgekar, I.Pop, Viscous dissipation effects on unsteady free convective flow past an infinite vertical porous plate with variable suction, Int.J.Heat mass transfer, 17(1974)85-90

[12] A.Barletta, Laminar mixed convection with viscous dissipation in a verticalo channel,

Int.Heat mass transfer.41(1998)3501-3513.

[13] A.Barletta,and Rossidi schi, E. Hahne, K. Spindler, Analysis of the effect of viscous dissipationforaminarflowinstadium-shapedducts,Int.Comns.HeatMass transfer.28(2001)

449-459.

[14] Bulent Yesilata , Effect of viscous dissipation on polymeric flows between two rotating coaxial parallel discs, Int. Comns.J.Heat Mass transfer,29(2002)589-600.

[15] M.A.El. Hakein, Viscous dissipation effects on MHD free convection flow over a nonisothermal surface in a micropolar fluid,Int. Comns. Heat Mass transfer,27(2000)

581-590.

[16] C.Israel , A. Ogulu, V. B. Omubo-Pepple, Influence of viscous dissipation and radiation on unsteady MHD free-convection flow past an infinite heated vertical plate in a porous medium with time-dependentsuction _Int. J. Heat Mass transfer ,40, (2003)2305-2311.

[17] P.Raveendra nath, P.M.V.Prasad,D.R.V.Prasada Rao,Computational hydromagnetic mixed convective heat and mass transfer through a porous medium in non-uniformly heated vertical channel with heat sources and dissipation,Int.J.Computers and Mathematics with applications,(Accepted for publication).

[18]A.Barletta,D.A.Nield,Mixed convection with viscous dissipation and pressure work in a lid-driven square enclosures,*52(2009) 4244-4253.*

[19] C.Lei,J.C.Patterson,Unsteady natural convection in a triangular enclosure induced by surface cooling,Int.J.Heat Fluid Flow26(2005)307-321.

[20] C.Lei,J.C.Patterson, A direct stability analysis of a radiation-induced natural convection boundary layer in a shallow water, J.Fluid Mech.480(2003)161-184.

[21] C.Lei,J.C.Patterson, A direct three dimensional simulation of radiation induced natural convection in a shallow Wedge,Int.J.Heat Mass Transfer 46(2003)1183-1197.

[22] C.Lei,J.C.Patterson, Unsteady natural convection in a triangular enclosure induced by absorption of radiation,J.Fluid Mech.460(2002)181-209.

[23] T.P.Bendarz,C.Lei,J.C.Patterson,An experimental study of unsteady natural convection in a reservoir model cooled from the water surface,Exp.Therm.Fluid sci.32(2008) 844-856.

[24] P.Tomasz,Bednarz, Chengwang Lei,John c.Patterson, A numerical study of unsteady natural convection induced by iso-flux surface cooling in a reservoir model,52(2009)56-66.

[25] S.U.S.Choi,Enhancing thermal conductivity of fluids with nanoparticles,in:D.A.Singer,

H.P.Wang(Eds.),Development and Applications of Non-Newtonian Flows,FED-vol.231/MD-vol.66,ASME,New york,1995,pp.99-106.

[26]J.A.Eastman, S.U.S.Choi, W.Yu, L.J. Thompson,Anomalously increased effective thermal conductivity of ethylene glycol-based nanofluids containing copper nanoparticles,Appl.phys.Lett.78(2001)718-720.

[27]S.K.Das,N.putra,P.Thiesen,W.Roetzel,Temperature dependence of thermal condctivity enhancement for nano fluids,ASMEJ.Heat Transfer 125(2003) 567-574.

[28]S.P.Jang,S.U.S.Choi, The role of Brownian motion in the enhanced thermal conductivity of nanofluids,Appl.Phys.Lett.84(2004)4316-4318.

[29] J.Koo, C.Kleinstreuer, A new thermal conductivity model for nanofluids,J.Nanoparticles Res.6(2004)577-588.

[30] J.Kim, Y.T.Kang,C.K.Choi, Analysis of convective instability and heat transfer

characteristics of nanofluids,Phys.Fluids 16 (2004) 2395-2401.

[31] Y.Xuan,Q.Li, Investigation on convective heat transfer and flow features of nanofluids,ASME J.Heat Transfer 125 (2003) 151-155.

[32] D.Wen, Y.Ding , Experimental investigation into convective heat transfer of nanofluids at the entrance region under laminar flow conditions,Int.J.Heat Mass Transfer 47 (2004)

5181-5188.

[33] Y.Xuan, W.Roetzel,Conceptions for heat transfer correlation of nanofluids, Int.J.Heat Mass Transfer 43 (2000) 3701-3707..