F–Centroidal Mean Labeling of Some Graphs

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Abstract: *In this paper, we have discuss the F–centroidal Mean Labeling of some graphs. A function f is called an F–centroidal mean labeling of a graph G(V,E) with p vertices and q edges if* $f : V(G) \rightarrow \{1,2,3,..........., q+1\}$ *is injective and the induced function* $f^* : E(G) \rightarrow \{1,2,3,.........., q\}$ *defined as*

$$f^*(uv) = \left\lfloor \frac{2\left[f(u)^2 + f(u)f(v) + f(v)^2\right]}{3\left[f(u) + f(v)\right]} \right\rfloor,$$

for all $uv \in E(G)$*, is bijective. A graph that admits an F–centroidal mean labeling is called an F–centroidal mean graph.*

Keywords: **Labeling, *F*–centroidal mean labeling, *F*–centroidal mean graph.**

1. Introduction:

All graphs in this paper are finite, simple and undirected. Terms not defined here are used in the sense of [10]. Let *G(V, E)* be a graph with *p* vertices and *q* edges. For a detailed survey on graph labeling, we refer [9].

Durai Baskar and Arockiaraj defined the *F*-harmonic mean labeling [8] and discussed its meanness of some standard graphs. The concept of *F*-geometric mean labeling was introduced by Durai Baskar and Arockiaraj [7] and it was developed [6]. The concept of *F*-root square mean labeling was introduced by Arockiaraj, [3] and they studied the *F*-root square mean labeling of some standard graphs [4]. Durai Baskar and Manivannan were introduced *F*-heronian mean labeling [5]. Motivated by the works of so many authors in the area of graph labeling we introduced a new type of labeling called an *F*-centroidal mean labeling.

A function f is called an F–centroidal mean labeling of a graph *G(V,E)* with *p* vertices and *q* edges if $f : V(G) \rightarrow \{1,2,3,..........., q+1\}$ is injective and the induced function $f^* : E(G) \rightarrow \{1,2,3,.........., q\}$ defined as

$$f^*(uv) = \left\lfloor \frac{2\left[f(u)^2 + f(u)f(v) + f(v)^2\right]}{3\left[f(u) + f(v)\right]} \right\rfloor,$$

for all $uv \in E(G)$, is bijective. A graph that admits an ***F*–centroidal mean labeling** is called an ***F*–centroidal mean graph.**

In this paper we have discussed the *F*–centroidal measures of the graph $D(T_n)$, the graph $AD(T_n)$, the graph $A(Q_n)$, the graph $AD(Q_n)$, the graph $AD(Q_n)$, the graph SL_n.

Main Results:

Definition: 2.1

A double triangular snake is obtained from a path v_1, v_2,…….., v_n by joining v_i and v_{i+1} to a new vertex w_i, i = 1, 2, ……., n – 1 and to a new vertex u_i for i = 1, 2, ……. , n – 1.

Theorem: 2.2

The graph $D(T_n)$ $(n \geq 3)$ is an *F*-centroidal mean graph.

Proof:

Let $\{v_i, 1 \leq i \leq n; u_i, u_i', 1 \leq i \leq n-1\}$ be the vertices and $\{a_i, b_i, 1 \leq i \leq 2n-2, c_i, 1 \leq i \leq n-1\}$ be the edges which are denoted as in Figure 1.1.

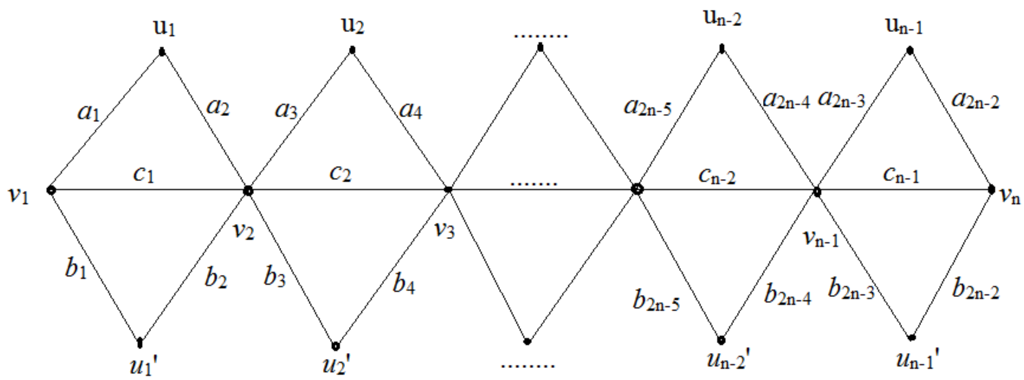

Figure 1.1: Ordinary labeling of $D(T_n)$

First we label the vertices as follows:

$f(v_1) = 1$; $f(v_2) = 4$

For $3 \leq i \leq n$, $f(v_i) = 5i - 4$

$f(u_1) = 2$; $f(u_2) = 10$

For $3 \leq i \leq n-1$, $f(u_i) = 5i - 3$

$f(u_1') = 6$; $f(u_2') = 8$

For $3 \leq i \leq n-1$, $f(u_i') = 5i - 1$

Then the induced edge labels are:

$$f^*(a_1)=1; f^*(a_2)=3; f^*(a_3)=7; f^*(a_4)=10$$

For $5 \le i \le 2n-2$, $f^*(a_i)=\begin{cases} \frac{5i-3}{2} & i \text{ is odd} \\ \frac{5i-2}{2} & i \text{ is even} \end{cases}$

$$f^*(b_1)=4; f^*(b_2)=5; f^*(b_3)=6; f^*(b_4)=9$$

For $5 \le i \le 2n-2$, $f^*(b_i)=\begin{cases} \frac{5i-1}{2} & i \text{ is odd} \\ \frac{5i}{2} & i \text{ is even} \end{cases}$

$$f^*(c_1)=2$$

For $2 \le i \le n-1$, $f^*(c_i)=5i-2$

Hence, f is an *F*-centroidal mean labeling of $D(T_n)$ $(n \ge 3)$.

F-centroidal mean labeling of $D(T_4)$ is shown in Figure 1.2.

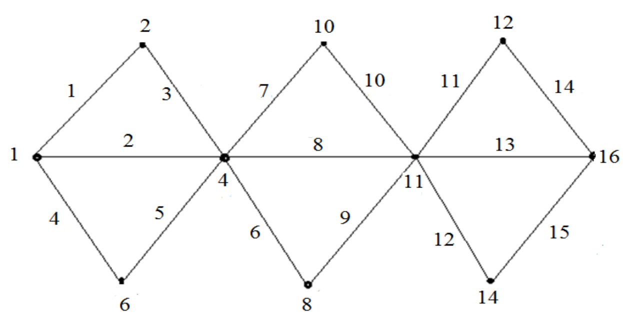

Figure 1.2: *F*-centroidal mean labeling of $D(T_4)$

Theorem: 2.3

The graph $AD(T_n)$ $(n \ge 4)$ is an *F*-centroidal mean graph.

Proof:

Let $\left\{u_i, u_i', 1 \le i \le \frac{n}{2}; v_i, 1 \le i \le n\right\}$ be the vertices and $\{a_i, c_i, 1 \le i \le n, b_i, 1 \le i \le 2n-1\}$ be the edges which are denoted as in Figure 1.3.

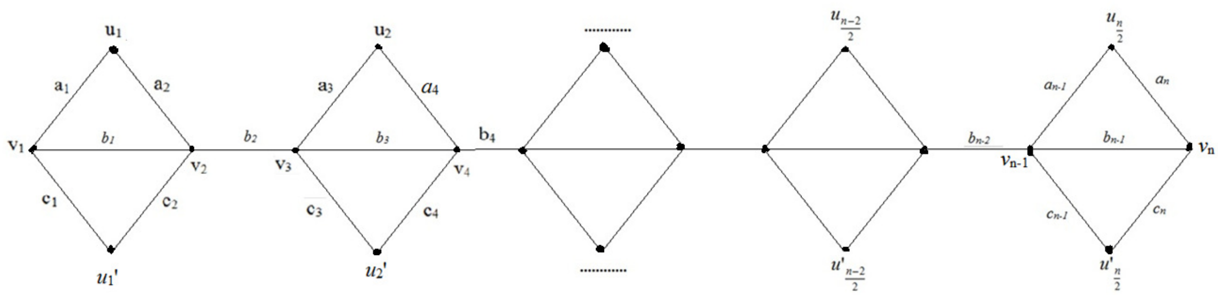

Figure 1.3: Ordinary labeling of AD(T_n)

First we label the vertices as follows:

$f(u_1) = 2$

For $2 \le i \le \frac{n}{2}$, $f(u_i) = 6i - 2$

For $1 \le i \le \frac{n}{2}$, $f(u_i') = 6i$

$f(v_1) = 1$

For $2 \le i \le n$, $f(v_i) = \begin{cases} 3i-2 & i \text{ is odd} \\ 3i-5 & i \text{ is even} \end{cases}$

Then the induced edge labels are:

For $1 \le i \le n$, $f^*(a_i) = 6i - 5$; i is odd.

$f^*(a_2) = 3$, For $3 \le i \le n$, $f^*(a_i) = 3i - 4$; i is even.

$f^*(b_1) = 2$; $f^*(b_2) = 5$

For $3 \le i \le n-1$, $f^*(b_i) = 3i$

For $1 \le i \le n$, $f^*(c_i) = \begin{cases} 3i+1 & i \text{ is odd} \\ 3i-1 & i \text{ is even} \end{cases}$.

Hence, f is an F-centroidal mean labeling ofAD(T_n) ($n \ge 4$).

F-centroidal mean labeling of $AD(T_6)$ is shown in Figure 1.4.

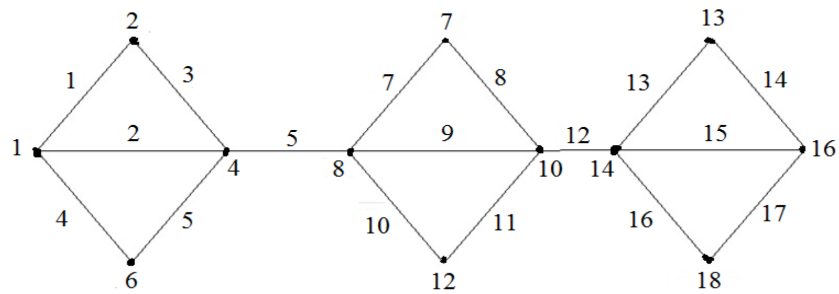

Figure 1.4: *F*-centroidal mean labeling of $AD(T_6)$

Definition: 2.4

An alternate quadrilateral snake $A(Q_n)$is obtained from a path $u_1, u_2,\ldots, u_n$ by joining u_i and u_{i+1} to a new vertices v_i, $w_i$respectively and then joining $v_i$andw_i. That is every alternative edge of a path replaced by a Cycle C_4.

Theorem: 2.5

The graph $A(Q_n)$ $(n \geq 2)$ is an *F*-centroidal mean graph.

Proof:

Let $\{u_i, v_i, 1 \leq i \leq n\}$ be the vertices and $\left\{a_i, 1 \leq i \leq \frac{n}{2}; b_i, 1 \leq i \leq n;\ c_i, 1 \leq i \leq n-1\right\}$ be the edges which are denoted as in Figure 1.5.

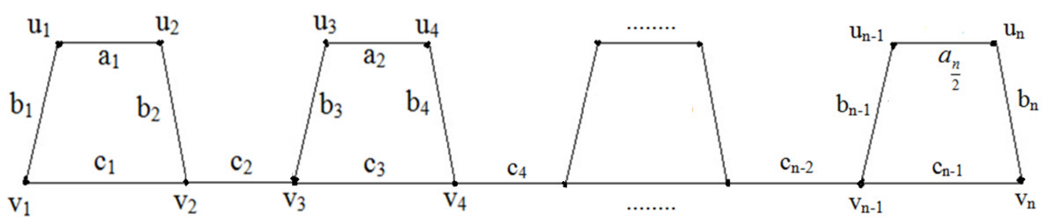

Figure 1.5: Ordinary labeling of $A(Q_n)$

First we label the vertices as follows:

$$\text{For } 1 \leq i \leq n,\ f(u_i) = \begin{cases} \dfrac{5i-1}{2} & i \ \text{is odd} \\ \dfrac{5i-4}{2} & i \ \text{is even} \end{cases}$$

For $1 \le i \le n$, $f(v_i) = \begin{cases} \frac{5i-3}{2} & i \text{ is odd} \\ \frac{5i}{2} & i \text{ is even} \end{cases}$

Then the induced edge labels are:

For $1 \le i \le \frac{n}{2}$, $f^*(a_i) = 5i-3$

For $1 \le i \le n$, $f^*(b_i) = \begin{cases} \frac{5i-3}{2} & i \text{ is odd} \\ \frac{5i-2}{2} & i \text{ is even} \end{cases}$

For $1 \le i \le n-1$, $f^*(c_i) = \begin{cases} \frac{5i+1}{2} & i \text{ is odd} \\ \frac{5i}{2} & i \text{ is even} \end{cases}$

Hence, f is an F-centroidal mean labeling of $A(Q_n)$ ($n \ge 2$).

F-centroidal mean labeling of $A(Q_6)$ is shown in Figure 1.6.

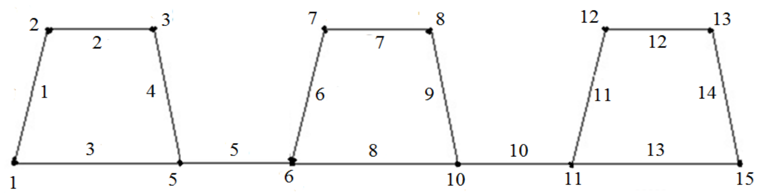

Figure 1.6: F-centroidal mean labeling of $A(Q_6)$

Theorem: 2.6

The Quadrilateral snake graph Q_n ($n \ge 2$) is an F-centroidal mean graph.

Proof:

Let $\{u_i, 1 \le i \le 2n-2; v_i, 1 \le i \le n\}$ be the vertices and $\{a_i, c_i, 1 \le i \le n-1; b_i, 1 \le i \le 2n-2\}$ be the edges which are denoted as in Figure 1.7.

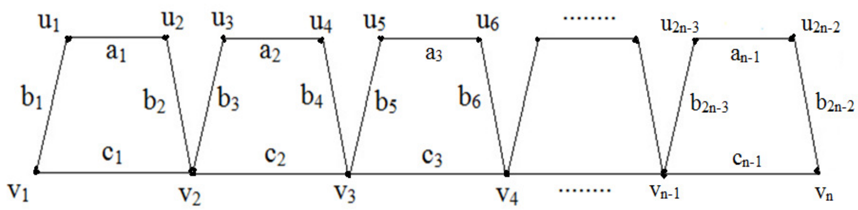

Figure 1.7: Ordinary labeling of Q_n

First we label the vertices as follows:

For $1 \le i \le 2n-2$, $f(u_i) = \begin{cases} 2i & i \text{ is odd} \\ 2i-1 & i \text{ is even} \end{cases}$

For $1 \le i \le n$, $f(v_i) = 4i-3$

Then the induced edge labels are:

For $1 \le i \le n-1$, $f^*(a_i) = 4i-2$.

For $1 \le i \le 2n-2$, $f^*(b_i) = \begin{cases} 2i-1 & i \text{ is odd} \\ 2i & i \text{ is even} \end{cases}$.

For $1 \le i \le n-1$, $f^*(c_i) = 4i-1$.

Hence, f is an *F*-centroidal mean labeling of Q_n $(n \ge 2)$

F-centroidal mean labeling of Q_4 is shown in Figure 1.8.

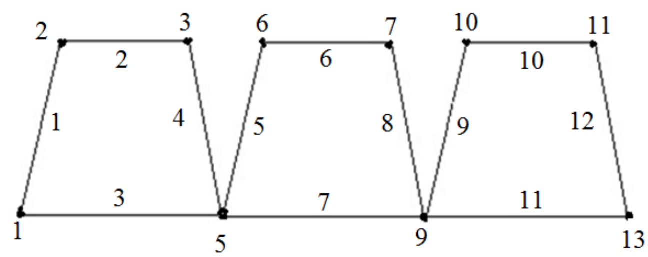

Figure 1.8: *F*-centroidal mean labeling of Q_4

Theorem: 2.7

The graph $AD(Q_n)$ $(n \ge 2)$ is an *F*-centroidal mean graph.

Proof:

Let $\{u_i, w_i, v_i, 1 \le i \le n\}$ be the vertices and $\left\{a_i, e_i, 1 \le i \le \frac{n}{2}; c_i, 1 \le i \le n-1, b_i, d_i, 1 \le i \le n\right\}$ be the edges which are denoted as in Figure 1.9.

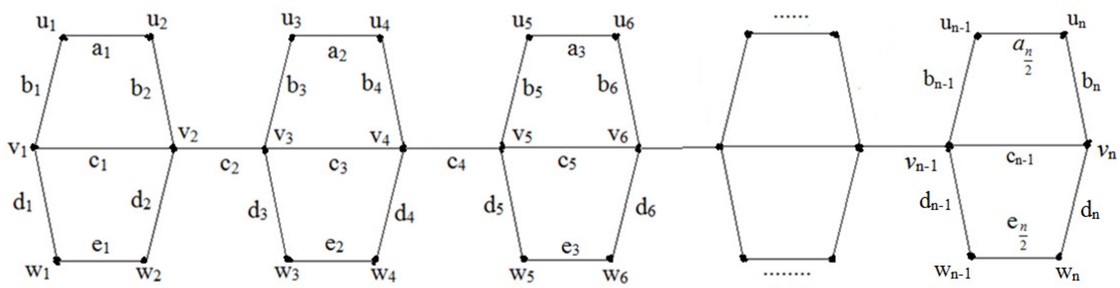

Figure 1.9: Ordinary labeling of AD(Q_n)

First we label the vertices as follows:

$f(u_1)=1, f(u_2)=5$

For $3 \le i \le n$, $f(u_i)=\begin{cases} 4i-2 & i \text{ is odd} \\ 4(i-1) & i \text{ is even} \end{cases}$

$f(v_1)=2, f(v_2)=7$

For $3 \le i \le n$, $f(v_i)=\begin{cases} 4i-3 & i \text{ is odd} \\ 4i & i \text{ is even} \end{cases}$

$f(w_1)=3, f(w_2)=8$

For $3 \le i \le n$; $f(w_i)=4i-1$

Then the induced edge labels are:

For $1 \le i \le \frac{n}{2}$, $f^*(a_i)=8i-5$.

For $1 \le i \le n$; $f^*(b_i)=\begin{cases} 4i-3 & i \text{ is odd} \\ 4i-2 & i \text{ is even} \end{cases}$.

For $1 \le i \le n-1$; $f^*(c_i)=4i$

For $1 \le i \le n$, $f^*(d_i)=\begin{cases} 4i-2 & i \text{ is odd} \\ 4i-1 & i \text{ is even} \end{cases}$.

For $1 \le i \le \frac{n}{2}$, $f^*(e_i)=8i-3$.

Hence, *f* is an *F*-centroidal mean labeling of AD(Q_n) ($n \ge 2$).

F-centroidal mean labeling of AD(Q_6) is shown in Figure 1.10.

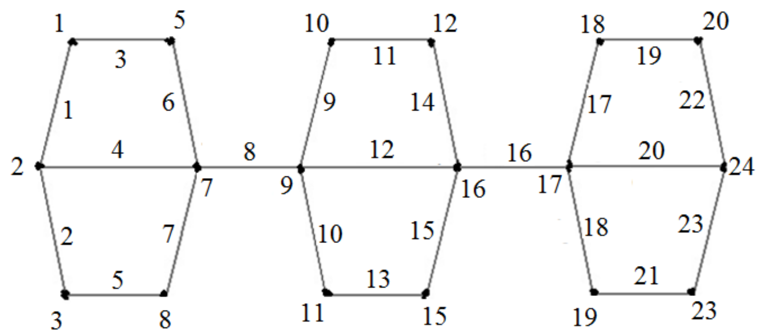

Figure 1.10: *F*-centroidal mean labeling of AD(Q_6)

Definition: 2.8

The slanting ladder SL_n is a graph obtained from two paths $u_1, u_2, \ldots, u_n$ and $v_1, v_2, \ldots, v_n$ by joining each u_i with v_{i+1}, $1 \le i \le n-1$.

Theorem: 2.9

The graph SL_n ($n \ge 2$) is an *F*-centroidal mean graph.

Proof:

Let $\{u_i, v_i, 1 \le i \le n\}$ be the vertices and $\{a_i, b_i, c_i, 1 \le i \le n-1\}$ be the edges which are denoted as in Figure 1.11.

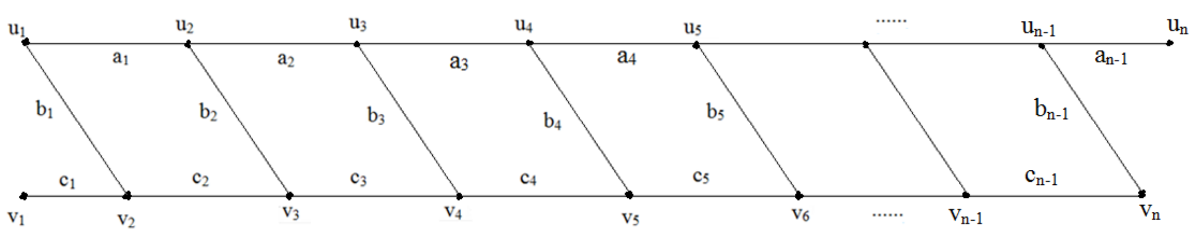

Figure 1.11: Ordinary labeling of SL_n

First we label the vertices as follows:

$$f(u_1) = 3$$

For $2 \le i \le n$, $f(u_i) = \begin{cases} 3i-2, & i \text{ is even} \\ 3i-1, & i \text{ is odd} \end{cases}$

$$f(v_1) = 1, f(v_2) = 2$$

For $3 \le i \le n$, $f(v_i) = 4(i-1)$.

Then the induced edge labels are:

For $1 \le i \le n-1$; $f^*(a_i) = 3i$.

For $1 \le i \le n-1$; $f^*(b_i) = 3i-1$.

For $1 \le i \le n-1$; $f^*(c_i) = 3i-2$

Hence, f is an F-centroidal mean labeling of SL_n (n≥ 2).

F-centroidal mean labeling of SL_6 is shown in Figure 1.12.

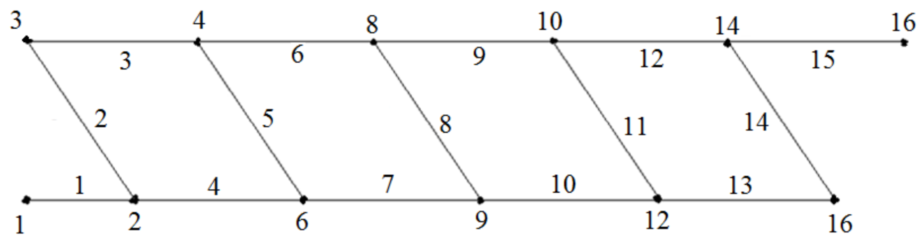

Figure 1.10: F-centroidalmean labeling of SL_6

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