STUDY ON SOME RESULT ON 2-METRIC SPACES

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ABSTRACT— The notion of 2 metric space lays introduced by Gahler. After the introduction of 2- metric space the space has been equipped with some important result of metric spaces. The notion of Cauchy references, convergent sequence and completeness have been introduced. Further fixed points for contradiction type mapping have also been obtained. The existence of a 2-metric is therefore of Paramount important in several areas of mathematics, physics and other sciences. The theory itself is a beautiful mixture of analysis (Pure and applied) topology and geometry. So It is very powerful and important trade in the study of non-linear phenomena.

KEYWORD— 2- metric , fixed point,Cauchy sequences etc.

The notion of 2-metric space is introduced in X in the following way. Let x be a non-empty and let d be a non-negative real valued function defined on X × X × X such that

(i) to each pair of point x, y ε X with x $\neq$ 0,

(ii) d (x, y, z) = 0 when at least two of the three points are equal

(iii) for any x, y, z ε X,

$$d(x, y, z) = d(x, y, z) = d(x, y, z)$$

(iv) for any x, y, w ε X,

$$d(x, y, z) \leq d(x, y, w) + d(x, w, z) + d(x, y, z).$$

Then (X, d) is called a 2-metric.

Unless or otherwise stated X stands for a 2-metric space with 2-metric d and N stands for the set of non-negative integers.

A sequence $\{x_n\}$ in X is called a Cauchy-sequence if

$$\lim_{m,n} \to \infty d(x_m, x_n, a) = 0 \text{ for all } a \,\varepsilon\, X.$$

A sequence $\{x_n\}$ in x is said to converge to x if

$$\lim_{m,n} \to \infty d(x_n, x, a) = 0 \text{ for all } a \,\varepsilon\, X.$$

There is an example to show that a convergent sequence need not be Cauchy which is not true in a metric space. A 2-metric space X is called complete it every Cauchy sequence in X is convergent.

A 2-metric d on X is said to be continuous if it is sequentially continuous in two of its arguments. It is know that a 2-metric d is continuous in any one of its arguments and that if it is continuous then it is continuous in all of its arguments. The ahs been observed that when d is continuous every convergent sequence is Cauchy but the converse need not be true.

Analogous to the notion of weak commutativity in metric space we have introduced the concept of weak commutativity in 2-metric space and we have an example that commuting pair of self maps are weakly commutative but the converse is not true. There are some common fixed point theorems using the concept of weakly commutative on x if d (fgs, gfx, a) $\leq$d (fx, gx, a) for all x, a εX and Semoniya pair of mappings in a 2-metric space which is alogous to the definition of compitable pair of mappings in metric space and there is a fixed point theorem in 2-metric spaces.

Let f and g be two self mappings on a 2-metric space X. f and g are called compitable or asymptotically commuting or z-asymptotically commuting if and only if

$$\lim_{n\to\infty} d(fgx_n, gfx_n, a) = 0$$

for all aεX and where $\{X_n\}$ is a sequence X such that

$$\lim_{n\to\infty} fx_n = \lim_{n\to\infty} gx_n = z.$$

It may be verified that weakly commuting mapping are compitable but the converse need not be true.

Very few literatures are available on fixed point of mappings in 2-metric spaces. Some of their results are cited below.

Theorem A Let X be a complete 2-metric space, $f : X \to X$ satisfying, there exists h, $0 < h < 1$ such that for each x, y, a, ε X.

d(f(x), f(y), a) $\leq$h max {d (x, y, a), d(x, f(x), a), d(y, f(y), a), d(x, f(y), a), d(f(y), x, a)}

They f possesses a unique point z and lim $f_n(x_0)$ = x for each $x_0 \varepsilon$ X.

Theorem B Let the mappings A, S, T : x $\to$X statisfying A (X) $\subset$ S(X) $\cap$ T(X), A commutes with S and T respectively and d(Ax, Ay, a) = h max {d (Sx, Ty, a), d(Ay, Ty, a), $\frac{1}{2}$ [d(Ax, Ty, a) + d(Ay, Sx, a)]}

for all x, y, a in X and h ε (0, 1). If S and T are continuous then A S and T have a unique common fixed point.

Let H be the set of all real valued functions.

$\varphi:[0,\infty)\to[0,\infty)$ such that each $\varphi \varepsilon$ H is non-decreasing upper semi continuous from right and φ (t) < t for t > 0.

Theorem C Let X be a 2-metric space with d continuous and self-mapping A, S, T on X. If there exist real numbers b_1, b_2, b_3, b_4 and b_5 such that $b_1 + 2b_2 + 2b_4 + b_5 < 1$

d (Ax, By, a) $\leq b_1$d (Sx, Sy, a) + b_2 {Tx, Ax, a) + b_3{d(Sx, Ax, a) + d(Ty, Ay, a)}

(i) +b_4{d(Sx, Tx, a) + d(Sy, Ty, a)}

b_5d (Ax, Ay, a) for all x, y ε X.

(ii) for a point x_0 in X there exists a sequence $\{X_n\}$ satisfying

$Sx_{2x+1} = Ax_{2x}$, $Tx_{2x+2} = Ax_{2n+1}$ and $Ax_{n+1} \neq Ax_{n+2}$, n = 0, 1, 2,….

(iii) the sequence $\{Ax_n\}$ has a subsequence converging to a point z in X.

(iv) A, S, T are continuous at z.

(v) {A, S} and {A, T} are z-asymptotically commuting then z is coincidence point of A, S, and T i.e. Az = Sz = Tz. If additionally $b_1 + b_4 + b_2 < 1$ then A, S and T have common fixed point which is unique also.

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