CFD Analysis of Convergent-Divergent Nozzle

***Neeraj Kumar Sharma[1]**

Animesh Kumar[2]

Anmol Sharma[3]

Bodh Prakash Singh[4]

Chirag Tyagi[5]

[1,2]*Assistant Professor,* [3,4,5] *B. Tech Student, Dept. of Mechanical Engineering, ABES EC, Ghaziabad.*
[1]*neeraj.sharma@abes.ac.in*
[2]*animesh.kumar@abes.ac.in*

Abstract: *This research has been carried out with an aim to introduce a more efficient and better performing nozzle for steam turbine applications. In an era of increasing Computational Fluid Dynamics related explorations in industries, the implementation of more efficient and higher performing nozzles will be essential. This project's objective is to bring these goals closer to reality. The objective includes design of different shapes of nozzles on the basis of reference data which had given a realistic effect to the analysis of these nozzle and also evaluated their performance for the same. CFD has given a realistic approach of testing of these nozzles by applying standard, constant and varying conditions to the different parameters of steam in fluent module through analysis results are obtained and evaluated which will be primary source for the further exploration in the design and analysis of nozzles. This will form the base for further evaluation in design and development of different shapes of nozzles.*

Keywords: *CD-Convergent Divergent, CFD-Computational Fluid Dynamics, NPR-Nozzle Pressure Ratio, FEM-Finite Element Method, FEA-Finite Element Analysis, IGES- Initial Graphics Exchange Specification.*

1. INTRODUCTION

This work aims at the design and analysis of CD nozzle with varying cross sections e.g., rectangular, square and circular and to determine which gives maximum outlet velocity. The analysis is done on the basis of the shape of the nozzles keeping input conditions, working fluid and material of nozzles same. The main purpose is to determine the best nozzle which offers highest outlet velocity compared to others.

1.1 Project Aims

The main aims of this project are as follows:

- Collection of coordinate data for design and modelling of cd nozzle. The coordinate data is to be collected from previous research papers and projects.
- Modeling the Supersonic Nozzle on SOLIDWORKS.
- Analysis of CD nozzle for supersonic nozzles of different cross section nozzles and meshing is to be carried out in ANSYS FLUENT.
- Analysis of result and readings obtained from analysis of different CD nozzles to determine which gives maximum exit velocity.

1.2 Steam Nozzle

The mass flow per second for wet steam

$$m = \frac{AV}{v} = \frac{AV}{x v_g}$$

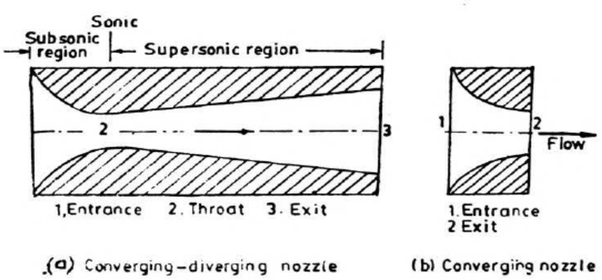


Figure1: Convergent Divergent Nozzle

Where,

A = Cross-sectional area in m²,
V = Steam velocity in m/sec,
v_s = Specific volume of dry saturated steam, m^3/kg,
x = Dryness fraction
$v = xv_s$ Specific volume of wet steam, m^3/kg.

From above equation, the cross-sectional area of nozzle will vary with the variation of $\frac{V}{xv_s}$ i.e., thus the product of A and $\frac{V}{xv_s}$ is constant.

1.3 Flow through Steam Nozzles:

According to thermodynamics, the flow of steam through nozzles may be considered as the process of adiabatic expansion.

1.4 Velocity of Steam Leaving Nozzle:

The kinetic energy gain is equal to the drop in enthalpy of the steam.
For isentropic flow :

$$\frac{V^2}{2} \times 1000 = H_1 \quad H_2 = H$$

Where H is enthalpy drop and V = exit velocity of steam

$$\therefore V = \sqrt{2 \times 1000H} = 44.72\sqrt{H} \text{ m/sec}$$

1.5 Mass of Steam Discharged:

The mass flow of steam can be written as

$$m = \frac{AV_2}{v_2}$$

$$\text{But } v_2 = v_1\left(\frac{p_1}{p_2}\right)^{\frac{1}{n}} = v_1\left(\frac{p_2}{p_1}\right)^{-\frac{1}{n}}$$

Where, v_1 =specific volume of steam at pressure p_1.

Using the value of V from equation

$$m = \frac{A}{v_2}\sqrt{\left[\frac{2000n}{n-1}(p_1v_1 - p_2v_2)\right]} = \frac{A}{v_2}\sqrt{\left[\frac{2000n}{n-1}\right]p_1v_1\left(1 - \frac{p_2v_2}{p_1v_1}\right)}$$

Putting the values of v_2 from eqn. we get,

$$m = \frac{A}{v_1\left(\frac{p_2}{p_1}\right)^{-\frac{1}{n}}}\sqrt{\left[2000\frac{n}{n-1}p_1v_1\left(1-\left(\frac{p_2}{p_1}\right)^{\frac{n-1}{n}}\right)\right]}$$

$$m = A\sqrt{\left[2000\frac{n}{n-1}\times\frac{p_1}{v_1}\left(\left(\frac{p_2}{p_1}\right)^{\frac{2}{n}}-\left(\frac{p_2}{p_1}\right)^{\frac{n+1}{n}}\right)\right]}$$

1.6 Critical Pressure Ratio

The rate of mass flow

$$m = A\sqrt{\left[2000\frac{n}{n-1}\times\frac{p_1}{v_1}\left(\left(\frac{p_2}{p_1}\right)^{\frac{2}{n}}-\left(\frac{p_2}{p_1}\right)^{\frac{n+1}{n}}\right)\right]}$$

The only variable in this equation is the ratio $\frac{p_2}{p_1}$.

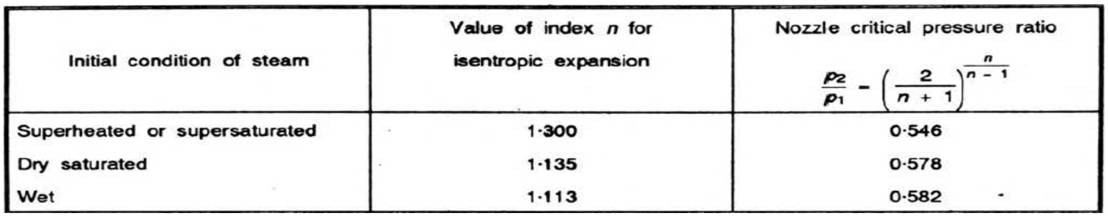
Hence, $\frac{m}{A}$ is maximum when $\left[\left(\frac{p_2}{p_1}\right)^{\frac{2}{n}}-\left(\frac{p_2}{p_1}\right)^{\frac{n+1}{n}}\right]$ is maximum.

Differentiating the above equation with respect to $\frac{p_2}{p_1}$ and equating to zero ,we get

$$\frac{p_2}{p_1} = \left[\frac{2}{n+1}\right]^{\frac{n}{n-1}}$$

$\frac{p_2}{p_1}$ is called as critical pressure ratio and depends on the index n.

The following values of index n and corresponding values of critical pressure ratios may be noted:



Initial condition of steam	Value of index *n* for isentropic expansion	Nozzle critical pressure ratio $\frac{p_2}{p_1} = \left(\frac{2}{n+1}\right)^{\frac{n}{n-1}}$
Superheated or supersaturated	1·300	0·546
Dry saturated	1·135	0·578
Wet	1·113	0·582

Table 1: Nozzle Pressure Ratio for Wet, Dry and Superheated Steam

The equation $\frac{p_2}{p_1} = \left[\frac{2}{n+1}\right]^{\frac{n}{n-1}}$ gives the ratio between the throat pressure p_2 and the inlet pressure p_1 for a maximum discharge per unit area through the nozzle.

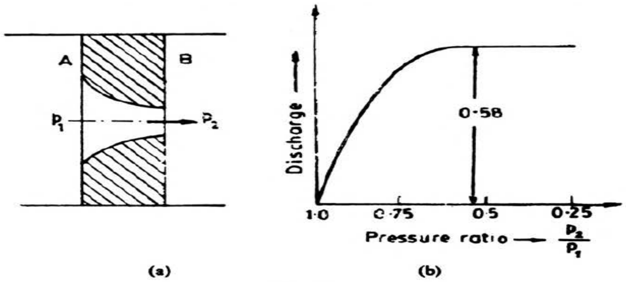


Figure 2: Graph between discharge and pressure ratio

1.7 Areas of Throat and Exit for Maximum Discharge

The first step is to calculate the critical pressure for the initial condition of steam. Using the Mollier chart, the drop in enthalpy is found by drawing a vertical line which represents the isentropic expansion from pressure p_1 to p_2.

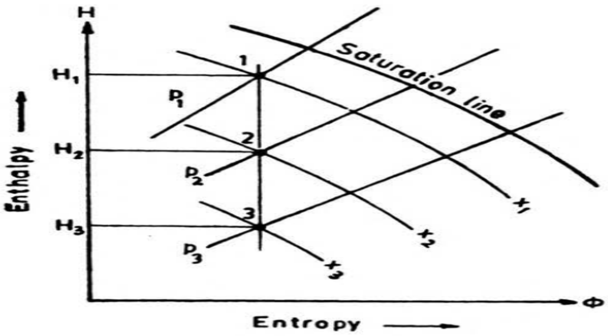


Figure 3: Mollier chart

Read off from the H - Φ chart the value of enthalpies H_1 and H_2 or enthalpy drop

$H_1 - H_2$ and dryness fraction x_2 as shown in fig 3.

Then, for throat, enthalpy drop from entry to throat, $H_t = H_1 - H_2$ kJ/kg, and velocity at throat, $V_2 = 44.72\sqrt{H_t}$ m/sec.

Then, mass flow, $m = \frac{A_2 V_2}{x_2 v_{s2}}$ kg/sec. (if steam is wet at throat)

Where A_2 = throat area.

If the steam is superheated at throat, $m = \frac{A_2 V_2}{v_{sup}}$ kg/sec.

The value of enthalpy H_3 and the dryness fraction x_3 at exit are read off directly from the H - Φ chart. For the exit or mouth of the nozzle, enthalpy drop from entry to exit:

$H_e = H_1 - H_3$ KJ/kg and velocity at exit, $V_3 = 44.72\sqrt{H_e}$ m/sec.

Then, the rate of mass flow, $m = \frac{A_3 V_3}{x_3 v_{s3}}$ kg/sec. The value of v_{s3} at p_3 can be found using from the steam tables. Now the exit area A_3 can be calculated by using above eqn.

1.8 Length of Nozzle

Linear (cone) contour of the de Laval nozzle are simple to manufacture, because it has constant angle α for whole length part t-e. The CD nozzles with such contour are used for supersonic blade row of one stage turbine.

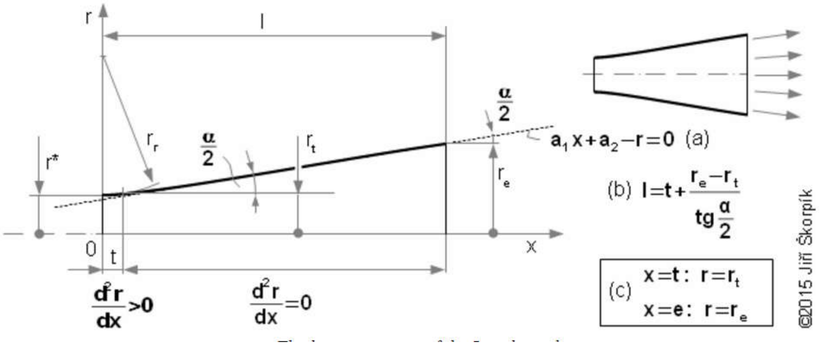


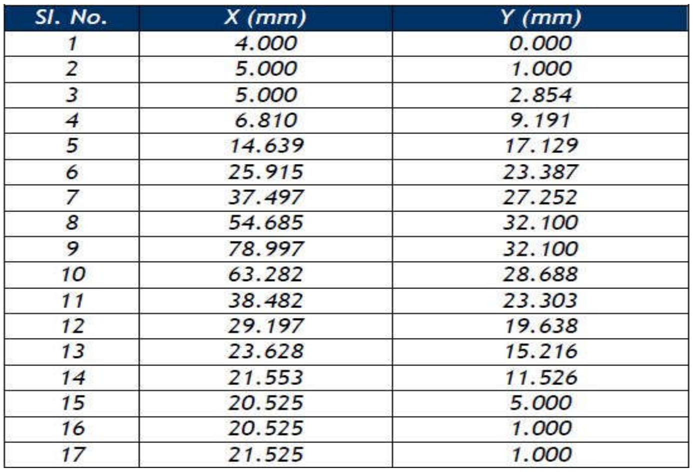
Figure 4: Linear contour of CD nozzle

(a) Equation of contour of nozzle for interval t-e; (b) equation of length of nozzle; (c) boundary conditions for calculation of a1, a2.

2. MODELING OF REFERENCE NOZZLE

2.1 Coordinates of Rectangular Bent Nozzle: Design of supersonic nozzle is done using coordinates .The below given coordinates are of a rectangular bent supersonic nozzle.

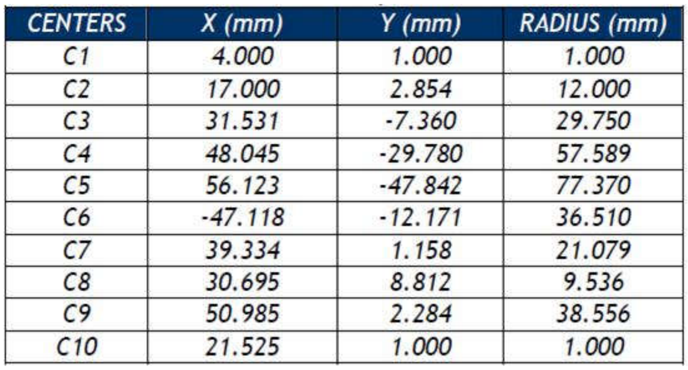
2.2 Coordinates for End Points of Rectangular Bent Nozzle



Sl. No.	X (mm)	Y (mm)
1	4.000	0.000
2	5.000	1.000
3	5.000	2.854
4	6.810	9.191
5	14.639	17.129
6	25.915	23.387
7	37.497	27.252
8	54.685	32.100
9	78.997	32.100
10	63.282	28.688
11	38.482	23.303
12	29.197	19.638
13	23.628	15.216
14	21.553	11.526
15	20.525	5.000
16	20.525	1.000
17	21.525	1.000

Table 2: Coordinates for End Points for Rectangular Bent Nozzle

2.3 Coordinates for Arc centers of Rectangular Bent Nozzle



CENTERS	X (mm)	Y (mm)	RADIUS (mm)
C1	4.000	1.000	1.000
C2	17.000	2.854	12.000
C3	31.531	-7.360	29.750
C4	48.045	-29.780	57.589
C5	56.123	-47.842	77.370
C6	-47.118	-12.171	36.510
C7	39.334	1.158	21.079
C8	30.695	8.812	9.536
C9	50.985	2.284	38.556
C10	21.525	1.000	1.000

Table3 : Coordinates for Arc centre of Rectangular Bent Nozzle

2.4 Design of Rectangular Bent Nozzle

From the data given in above tables coordinates of end point of nozzle profile and coordinates of arc center are drawn in SOLIDWORKS. Then a 2D model of rectangular bent nozzle is created in SOLIDWORKS.

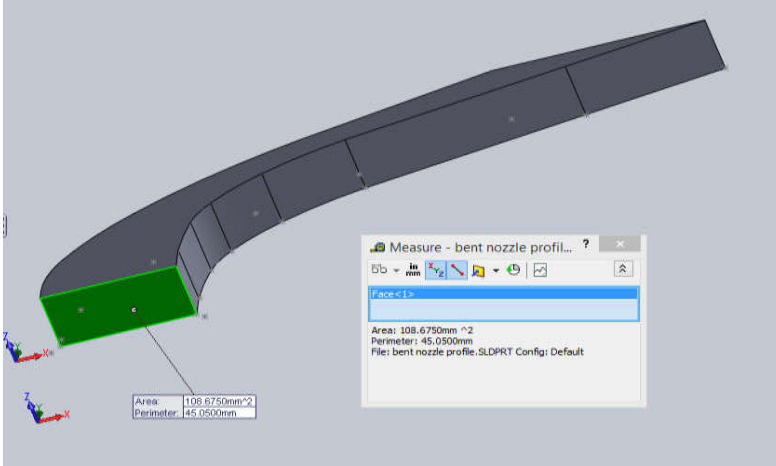
2.5 Reference Areas and Axial Length

Using measure tool in SOLIDWORKS the inlet area, throat area and axial length of rectangular bent supersonic nozzle is evaluated.

Inlet area = 108.675 mm^2

Throat area = 28.5 mm^2

Axial length = 68.7 mm



Area: 108.6750mm ^2
Perimeter: 45.0500mm
File: bent nozzle profile.SLDPRT Config: Default

Figure 5: Solid Model of Rectangular Bent Nozzle

3. CALCULATIONS FOR DESIGN OF NOZZLES

3.1 Parameters and Calculation:

Parameters of Steam: The following parameters of working substance (steam) are shown as:

Sr. No.	Properties	Value	Unit
1	Pressure	130	Bar
2	Superheat Temperature	535	℃
3	Saturation Temperature	331.493	℃
4	Degrees Superheat	203.507	℃
5	Specific Enthalpy of Evaporation (h_{fg})	1.12454e+06	J/kg
6	Specific Enthalpy of Superheated Steam (h)	3.42928e+06	J/kg
7	Density of Steam	38.5699	Kg/m^2
8	Specific Volume of Steam (v)	0.0259269	m^2/kg
9	Specific Heat of Steam (c_p)	1006.43	J/Kg-K
10	Speed of sound	540	m/s
11	Dynamic Viscosity of Steam	3.06134e-05	N-s/m^2

Table 4: Parameters of Steam

3.2 Critical Pressure: The critical pressure for the known initial condition of steam.

$$\frac{p_2}{p_1} = \left[\frac{2}{n+1}\right]^{\frac{n}{n-1}}$$

Since n = 1.3

$$\frac{p_2}{p_1} = \left[\frac{2}{1.3+1}\right]^{\frac{1.3}{1.3-1}} = 0.546$$

$$p_2 = 70.98 \text{ bar}$$

3.3 Enthalpy: From the Mollier chart, the enthalpy drop can be found using the isentropic expansion from $p_1 = 130\ bar$ to $p_2 = 70.98\ bar$ (p_2 is throat pressure).
From the H - Φ chart

$H_1 = 3440$ KJ/kg and
$H_2 = 3250$ KJ/kg

The enthalpy drop from entry to throat,

$H_t = H_1 - H_2 = 3440 - 3250 = 190$ KJ/kg

From Mollier (H - Φ) chart

$v_{sup\,2} = 0.02 + 0.0255 = 0.0455\ \text{m}^3/\text{Kg}$

3.4 Velocity at Throat

Velocity at throat, $V_2 = 44.72\sqrt{H_t} = 44.72\sqrt{190} = 616.42$ m/sec.

3.5 Mass Flow Rate

$m = \frac{A_2 V_2}{v_{sup2}}$ Kg/sec. (as the steam is superheated at throat)

We already know throat area which is equal to that of throat area of rectangular bent nozzle

Throat area = $A_2 = 2.85 \times 10^{-5}\ m^2$ $m = \frac{2.85 \times 10^{-5} \times 616.42}{0.0455} = 0.386$ Kg/sec

3.6 Area at Exit

Since the back pressure in this nozzle is less than the critical pressure, the vertical line on the Mollier chart is extended up to p_3 at the exit. The value of enthalpy H_3, specific volume $v_{sup\,3}$ and temperature T_3 at exit are read off directly from the Mollier chart.

$H_3 = 2915$ KJ/kg
$v_{sup\,3} = 0.11428\ \text{m}^3/\text{Kg}$
$T_3 = 255.55$ ℃

For the exit or mouth of the nozzle, enthalpy drop from entry to exit,

$H_e = H_1 - H_3 = 3340 - 2915 = 525$ KJ/kg and ,
$V_3 = 44.72\sqrt{H_e} = 44.72\sqrt{525} = 1024.66$ m/sec

Then, mass flow,

$m = \frac{A_3 V_3}{x_3 v_{sup3}}$ Kg/sec

As the mass of discharge m is known, the exit area A_3 can be calculated by using above eqn.

$$A_3 = \frac{mv_{sup3}}{V_3} \quad , \quad A_3 = \frac{0.386 \times 0.11428}{1024.66}$$

$$A_3 = 43.05\ mm^2$$

3.7 Calculation for Length of Nozzle

The axial length of nozzles is taken same as that of rectangular bent supersonic nozzle.

We know that Mach no. and degree of expansion depends on divergence angle of a nozzle. Hence axial length and divergence angle is kept constant for all the nozzles. The distance of throat from inlet is calculated for each nozzle.

We know,

$$L = T + \frac{R_e - R_t}{\tan\frac{\alpha}{2}}$$

Where, T = location of throat from inlet, L = axial length = 68.7 mm (as determined previously), α = angle of divergence, Taking $\alpha = 3^{\circ}$(angle of divergence of each nozzle is taken 3 °).

3.8 For circular nozzle:

$$L - T + \frac{R_e - R_t}{\tan\frac{\alpha}{2}} \quad 68.7 = T + \frac{3.7 - 3.01}{\tan\frac{3}{2}} \quad T = 38.7\ mm$$

3.9 For square nozzle:

$$L = T + \frac{L_e - L_t}{\tan\frac{\alpha}{2}} \quad 68.7 = T + \frac{6.56 - 5.338}{\tan\frac{3}{2}} \quad T = 45.366\ mm$$

3.10 For rectangular nozzle:

$$L = T + \frac{L_e - L_t}{\tan\frac{\alpha}{2}} \quad 68.7 = T + \frac{6.15 - 4.07}{\tan\frac{3}{2}} \quad T = 28.984\ mm$$

4. RESULT & DISCUSSION

After analysis in ANSYS FLUENT 16.0, the contours for temperature, velocity and pressure are generated and variation in them has been studied. Comparison of contours of Mach number, pressure and temperature of nozzles of different cross sections is done.

4.1 Mach Number Contours

4.1.1 Mach No. Contour for Circular Nozzle: By observing the Mach no. contour of circular nozzle. We found that maximum Mach no. at exit of circular nozzle is 1.88.

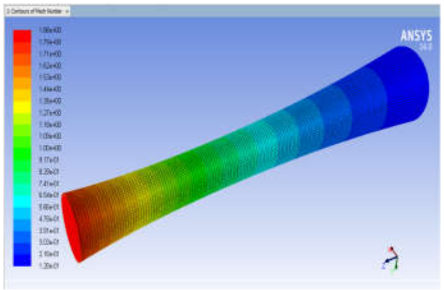


Figure 6: Mach number Contour for Circular nozzle

4.1.2 Mach No. Contour for Square Nozzle: By observing the Mach no. contour of square nozzle. We found out that the maximum Mach no. at exit of square nozzle is 1.85.

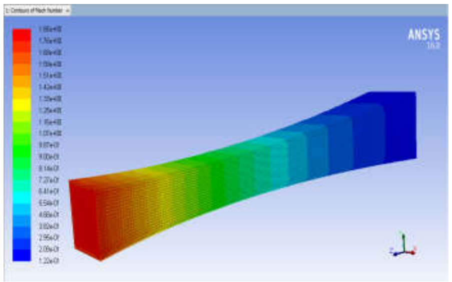


Figure 7: Mach number Contour for Square nozzle

4.1.3 Mach No. Contour for Rectangular Nozzle: By observing the Mach no. contour of rectangular nozzle.

We found out that the maximum Mach no. at exit of rectangular nozzle is 2.16.

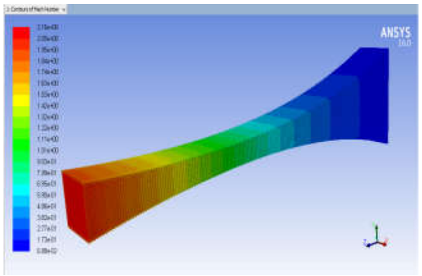


Figure 8: Mach number Contour for Rectangular nozzle

4.2 Pressure Contours

4.2.1 Pressure Contour for Circular Nozzle: By observing the pressure contour of circular nozzle. We found out that the maximum pressure drop through the circular nozzle is 10.96e+06 Pa.

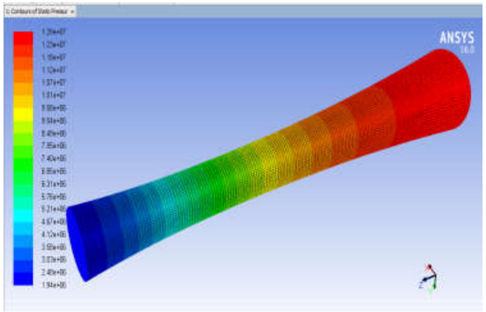


Figure 9: Pressure Contour for Circular nozzle

4.2.2 Pressure Contour for Square Nozzle: By observing the pressure contour of square nozzle. We found out that the maximum pressure drop through the square nozzle is 10.9e+06 Pa.

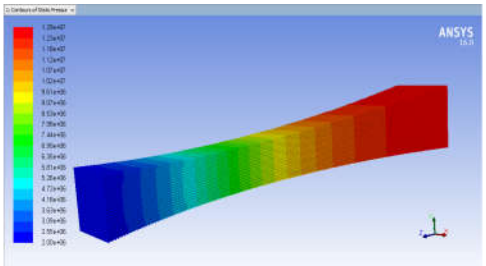


Figure 10: Pressure Contour for Square nozzle

4.2.3 Pressure Contour for Rectangular Nozzle: By observing the pressure contour of rectangular nozzle. We found out that the maximum pressure drop through the rectangular nozzle is 11.72e+06 Pa.

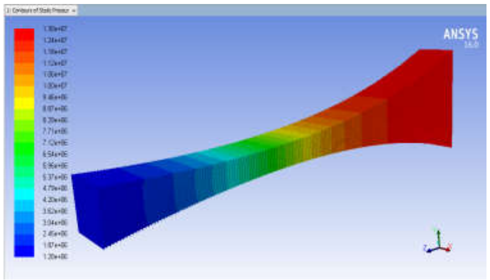


Figure 11: Pressure Contour for Rectangular nozzle

4.3 Temperature Contour

4.3.1 Temperature Contour for Circular Nozzle: By observing the temperature contour of circular nozzle. We found out that the maximum temperature drop through the circular nozzle is 286 K.

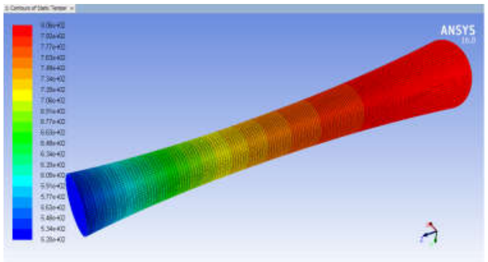


Figure 12: Temperature Contour for Circular nozzle

4.3.2 Temperature Contour for Square Nozzle: By observing the temperature contour of square nozzle. We found out that the maximum temperature drop through the square nozzle is 280 K.

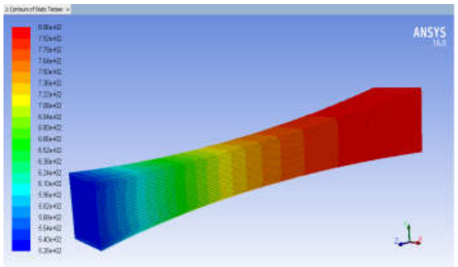


Figure 13: Temperature Contour for Square nozzle

4.3.3 Temperature Contour for Rectangular Nozzle: By observing the temperature contour of rectangular nozzle. We found out that the maximum temperature drop through the rectangular nozzle is 342 K.

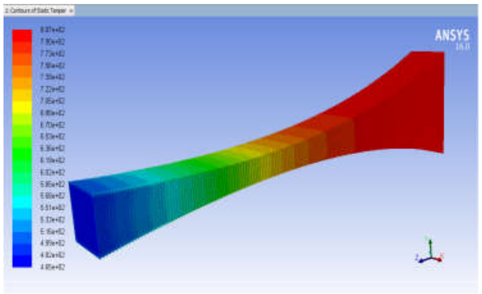


Figure 14: Temperature Contour for Rectangular nozzle

5. CONCLUSION

On the basis of the comparison we discovered that compared to nozzle with square and circular cross section, a nozzle with rectangular cross section gives velocity rise of about 16.75% and 14.89% respectively and a rise in pressure drop of 36% and 34.02% respectively and an increase in temperature drop of 11.59% and 10.58% respectively.

It can thus be concluded that the fluid properties like temperature, velocity and pressure largely depends on the nozzle cross-sectional area, which affect the flow inside the nozzle and the extent of flow expansion.

REFERENCES

[1] *R. C. Patel, C. J. Karamchandani, Steam Nozzles, Elements of Heat Engines, Volume II-Sixteenth Edition, Acharya Book Depot, Baroda (1963)*

[2] *Flow of gases and steam through nozzles. Web: http://www.transformacni-technologie.cz/en_40.html, by J Škorpík (2006)*

[3] *Bhaskar Roy, A.M. Pradeep,Jet Aircrafct Propulsion, Department Of Aerospace Engineering IIT Bombay, NPTEL,Lecture-31.*

[4] *B.Krishna Prafulla, Dr. V. Chitti Babu and Sri P. Govinda Rao, CFD Analysis of Convergent- Divergent Supersonic Nozzle, International Journal of Computational Engineering Research, Vol-03,Issue-5 (2013)*

[5] *A.K. Mohanty, Introduction to Compressible Flows, Fluid Mechanics, Prentice Hall of India (2006)*

[6] *David C.Wil Cox, "Turbulence modeling for CFD" Second Edition (1998).*

[7] *H.K.Versteeg and W.Malala Sekhara, "An introduction to Computational fluid Dynamics", British Library cataloguing pub, 4th edition (1996)*

[8] *Prosun Roy, Abhijit Mondal, Biswanath Barai, CFD Analysis of Rocket Engine Nozzle, International Journal of Advanced Engineering Research and Science (IJAERS) Vol-3, Issue-1, ISSN: 2349-6495 (2016)*

[9] *Lars Davidson, "An introduction to turbulence Models", Department of thermo and fluid dynamics, Chalmers university of technology, Goteborg, Sweden (2003)*

[10] *Gutti Rajeswara Rao, U.S. Ramakanth, A. Lakshman, Flow Analysis in a Convergent-Divergent Nozzle Using CFD (2013)*

[11] *B.Krishna Prafulla, V. Chitti Babu and P. Govinda Rao, CFD Analysis of Convergent-Divergent Supersonic Nozzle, International Journal of Computational Engineering Research, Vol-03, Issue-5 (2013)*

[12] *Samynathan.R, Durairaj.S, Manikandan.M, Balaji.E , Design and CFD analysis of flow through C-D nozzle and Aerospike nozzle at different operating conditions, International Journal of Innovative Research in Technology, Science & Engineering (IJIRTSE), ISSN: 2395-5619, Volume – 2, Issue – 6 (2016)*

[13] *C.Satheesh, A.Arulmurugu, Design and analysis of c-d nozzle increase the efficiency using CFD, International Journal of Modern Trends in Engineering and Research, e-ISSN: 2349-9745, p-ISSN: 2393-8161 (2015)*

[14] *A.A.Khan and T.R.Shem bharkar, "Viscous flow analysis in a Convergent-Divergent nozzle". Proceedings of the international conferece on Aero Space Science and Technology, Bangalore, India (2008)*

[15] *P. Parthiban, M. Robert Sagayadoss and T. Ambikapathi 'Design and Analysis of Rocket Engine Nozzle by using CFD and Optimization of Nozzle Parameters'. International Journal of Engineering Research. Vol 3, Issue 5 (2015)*

[16] *Biju Kuttan P and M Sajesh, 'Optimization of Divergent Angle of a Rocket Engine Nozzle using Computational fluid Dynamics'. The International Journal of Engineering and Science (IJES). Vol 2, Issue 2, pp 196-207 (2013)*

[17] *Pandey K.M.; Singh A.P.; CFD Analysis of Conical Nozzle for Mach 3 at Various Angles of Divergence with Fluent Software, International Journal of Chemical Engineering and Applications, Vol. 1, No. 2, ISSN: 2010-0221 (2010)*

[18] *Natta, Pardhasaradhi.; Kumar, V.Ranjith.; Rao, Dr. Y.V. Hanumantha.; Flow Analysis of Rocket Nozzle Using Computational Fluid Dynamics (CFD), International Journal of Engineering Research and Applications (IJERA), ISSN: 2248-9622,Vol. 2, Issue 5, pp.1226-1235 (2012)*

[19] *P. Padmanathan, Dr. S. Vaidyanathan, Computational Analysis of Shockwave in Convergent Divergent Nozzle, International Journal of Engineering Research and Applications (IJERA), ISSN: 2248-9622 , Vol. 2, Issue 2, pp.1597-1605 (2012)*

[20] *Anderson, John D.Jr.; Modern Compressible Flow with Historical Perspective, Third edition (2012)*

[21] *Vipul Sharma, Ravi Shankar, Gaurav Sharma, - CFD Analysis of Rocket Nozzle – University of Petroleum and energy studies, Department of Aerospace Engineering, Dehradun, India (2011)*

[22] *Mayur Chakravarti - Design analysis of flow in convergent divergent rocket nozzle – University of Petroleum and energy studies, Department of Aerospace Engineering, Dehradun, India (2011)*

[23] *Leonid Moroz, Yuri Gororuschenko, Petr Pagur, "Axial turbine stages design: ID/2D/3D simulation, Experimental, Optimization", Proceedings of GT 2005 ASME Turbo Expo, Nerada USA (2005)*

[24] *K. Pougatch, M. Salcudean, E. Chan, B. Knapper (2008) Modelling of compressible gas-liquid flow in a convergent-divergent nozzle Chemical Engineering Science, 63, 4176-4188 (2008)*

[25] *Kunal Pansari, S.A.K Jilani, Analysis of The Performance and Flow Characteristics of Convergent-Divergent Nozzle Department of mechanical Engineering (2013)*

[26] *Menandro S. Berana, Determination of Efficiency of Converging-Diverging Nozzles, Transcritical and Transonic Flow of CO2 Proceedings of the World Congress on Engineering, Vol II (2014)*

[27] *S. Vahaji, A.A. Akbarzadeh, A.Date, S.C.P Cheung, J.Y. Tu (2014) Efficiency of a two phase nozzle for geothermal power generation Applied Thermal Engineering 73, 227-235 (2014)*