GROUP ACTION ON prw-CONNECTEDNESS IN TOPOLOGY

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MSC : 54A05, 54D05, 54H11.

Abstract: *The aim of this paper to introduce the new type of connected spaces called group action on prw-connected spaces in topological spaces and also we obtain some properties of such spaces.*

Keywords: **Group acting on prw -open sets, prw-separated, prw -connectedness.**

1. INTRODUCTION

Connectedness [1] is a well-known notion in topology. Many researchers have investigated the basic properties of connectedness. Connectedness in [6–8] are used to expand some topological spaces. Connectedness are powerful tool in topology but they have many dissimilar properties. The notion of connectedness and compactness are useful and fundamental notions of not only general topology but also of other advanced branches of mathematics. D. Andrijevic [2] introduced a new class of generalized open sets in a topological space called b-open sets. The class of b-open sets generates the same topology as the class of b-open sets. Since the advent of this notion, several research paper with interesting results in different respects came into existence. M. Ganster and M. Steiner [5] introduced and studied the properties of gb-closed sets in topological spaces. In 1974, Das [4] defined the concept of semi-connectedness in topology and investigated its properties. Benchalli et al [3] introduced gb - connectedness in topological spaces. Later, Shibani [9] introduced and analyzed rg - connectedness. Vadivel et al [12] studied $rg\alpha$- interior and $rg\alpha$ - closure sets in topological spaces. In this paper, we introduce and investigate on group acting on prw -connectedness in topological spaces.

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2. PRELIMINARIES

The whole paper, Group (G, τ) (or simply G) always means a topological group on which no separation axioms are assumed unless explicitly stated.
We require the following definitions.

Definition 2.1: A topological group is a set G with two structures : (i). G is a group, and (ii) G is a topological space, Such that the two structures are compatible. (i.e)., the multiplication map $\mu : G\times G\longrightarrow G$ and the inversion map $v : G\longrightarrow G$ are both continuous.

Definition 2.2: Two subsets A and B of a group G is called separated if $cl(A) \cap B = \emptyset$ and $A \cap cl(B) = \emptyset$.

Definition 2.3: A topological group (G,τ) is called connected if G cannot be expressed as the union of two non empty disjoint open sets. Or equivalently, a topological group (G,τ) is called connected if G cannot be expressed as the union of two non empty disjoint sets A and B satisfying $(A \cap cl(B)) \cup (cl(A) \cap B) = \emptyset$.

Definition 2.4: A topological group G is said to be gb-connected if G cannot be expressed as a disjoint of two non-empty gb-open sets in G. A sub set of G is gb-connected if it is gb-connected as a subspace.

Definition 2.5: A topological group (G,τ) acting on a subset M is said to be pre-regular weakly closed (briefly prw-closed) set if $pcl(M) \subseteq U$ whenever $M \subseteq U$ and U is rs-open in G. The complement of prw-closed in G is called prw-open set.

3. GROUP ACTION ON prw-CONNECTEDNESS IN TOPOLOGICAL SPACES

Definition 3.1. A topological group (G,τ) is said to be Pre-regular weakly connected (briefly prw-connected), if group G cannot be expressed as a disjoint union of two non-empty prw-open sets in G.

Theorem 3.2. A group G acting on every prw-connected space is connected.

Proof. Let G be prw-connected. Suppose G is not connected. Then, $G = M \cup N$, where M and N are disjoint non-empty open sets in G. Since every open set is prw-open, M and N are prw-open. Thus, $G =$

$M \cup N$, where M and N are disjoint non-empty prw-open sets in G. This contradicts G is prw-connected. Hence, G is connected.

The Converse need not be true as seen in the following example.

Example 3.3. Let $G = \{ 1 , 3 , 7 , 9 \}$ be a group acting on a topology $\tau = \{ \emptyset , G \}$. Then G is connected space but not prw-connected space.

Theorem 3.4. A topological group (G, τ) acting on prw-separated sets M and N and if (G', τ') is an prw-connected subspace of (G, τ) then G' lies entirely within M or N.

Proof. Since M and N are prw-open in G, the sets $M \cap G'$ and $N \cap G'$ are prw-open in G'. These two sets are disjoint and their unions in G'. If they were both non-empty, they would be prw-separated sets of G'. Thus, one of them is empty. Hence, G' must lie entirely in M or N.

Theorem 3.5. Let M be a prw-connected subspace of group G. If $M \subseteq N \subseteq prw\text{-}cl(M)$ then N is also prw-connected.

Proof. Let M be a prw-connected and let $M \subseteq N \subseteq prw\text{-}cl(M)$. Suppose $N = P \cup Q$ is a separation of N by prw-open sets. By using theorem 3.4, M must lie entirely P or Q. Suppose that $M \subseteq P$ then $prw\text{-}cl(M) \subseteq prw\text{-}cl(N)$. Since $prw\text{-}cl(P)$ and Q are disjoint. N cannot intersect Q. This contradicts, P is non-empty subset of N. So that $Q = \emptyset$ which implies N is prw-connected.

Definition 3.6. Let (G, τ) be a topological group and $M \subseteq G$. A point $x \in G$ is said to be an Pre-regular weakly adherent point (briefly prw- adherent) of M, if every prw-open set containing x , contains at least one point of M.

Theorem 3.7. A topological group (G, τ) acting on any two prw- open subsets are prw- separated iff they are disjoint.

Proof. Since, a group acting on prw-separated sets, prw-open separated sets are disjoint. Conversely, let M and N are two disjoint prw-open sets. Suppose, $M \cap prw\text{-}cl(N) \neq \emptyset$. Let $x \in M \cap prw\text{-}cl(N)$. Then, $x \in M$ and x is an prw- adherent point of N. Since M is an prw-open set containing x and x is an prw- adherent point of N. Thus, $M \cap N \neq \emptyset$. Its contradicts M and N are disjoint. Therefore, $M \cap prw\text{-}cl(N) = \emptyset$. Similarly $prw\text{-}cl(M) \cap N = \emptyset$. Hence M and N are prw-separated.

Theorem 3.8. A topological group (G, τ) is an prw-disconnected iff G is the union of two non-empty disjoint prw-open sets.

Proof. Necessity: Let G be prw-disconnected. Then, there exists a nonempty proper subset M of G which is both prw-open and prw-closed. Then its complement $(G - M)$ is also a nonempty proper subset of G that is both prw-open and prw-closed. Thus $G = M \cup M^c$ and $\emptyset = M \cap M^c$. Therefore, G is the union of two non-empty disjoint prw-open sets.

Sufficiency: Let G be the union of two non-empty disjoint prw-open sets M and N. Then $M = N^c$. Now, N is prw-open, M is prw-closed. Since N is non-empty, M is a non-empty proper subset of group G that is both prw-open and prw-closed. Thus, G is prw-disconnected.

Theorem 3.9. If (G, τ) is prw-disconnected and if $h : (G, \tau) \to (G', \tau')$ is prw-continuous onto then (G', τ') is connected.

Proof. Suppose G' is not connected. Let $G' = M \cup N$, where M and N are disjoint non-empty open sets in G'. Since h is prw-continuous and onto, $G = h^{-1}(M) \cup h^{-1}(N)$, where $h^{-1}(M)$ and $h^{-1}(N)$ are disjoint non-empty prw-open sets in G. This contradicts the fact that G is prw-connected. Hence G' is connected.

Theorem 3.10. A topological group (G, τ) acting on any two prw-closed subsets are prw-separated if and only if they are disjoint.

Proof. Necessity: Since, a group acting on prw-separated sets are disjoint. So that prw-closed separated sets are disjoint.

Sufficiency: Let M and N are two disjoint prw-closed sets. Then,
$prw\text{-}cl(M) = M$, $prw\text{-}cl(N) = N$ and $M \cap N = \emptyset$.
Consequently $M \cap prw\text{-}cl(N) = \emptyset$ and $prw\text{-}cl(M) \cap N = \emptyset$. Hence M and N are prw-separated.

4. CONCLUSION

In this paper, Group action on prw-connectedness in topological spaces are introduced and some results will be analyzed for group action on prw- connected properties.

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