On b-Open set in Topological Space

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Abstract : In this paper, we introduce a new class of open sets in the topological space, called b-open sets in the topological space and discussed some of the basic properties of these sets. These class of sets is contained in the class of semi-pre-open sets and contains all semi-open sets and all pre-open sets. It is proved that the class of b-open sets generates the same topology as the class of pre-open sets.

Keywords : open set, closed set, interior of a set, closure of a set.

1. Introduction :

The concept of the term b-open sets in the topological space was first introduced by the mathematician Dimitrije Andrijevic(1) in the year 1996. The concept of semi-open sets and semi-continuity in topological spaces is introduced by Levine in 1963. The pre-open set is defined and studied Mashhour et al. in the year 1982, there has been extensive study in the direction by a host of topologist and the concept has found applications in illuminating several situations in this area.

In this paper, we investigate the properties of b-open set in the topological space and to attempt to develop a topology based on b-open set.

2. Preliminaries :

Throughout this paper (X , T) is always denote topological spaces on which no separation axioms are assumed, unless otherwise mentioned. When the set A is a subset of (X , T) then $C_L(A)$ & $I_N(A)$ are denote the closure and interior set of A, respectively. We recall some known definition needed in this paper.

2.1 . b-open set [1] : Let (X , T) be a topological space of a nonempty set X with the topology T & let $A \subseteq X$ then the set A is said to be b-open set in the topological space iff $A \subset C_L [I_N (A)] \cup I_N [C_L(A)]$; i.e., a b-open set is a set which is contained in the union of $C_L [I_N (A)]$ & $I_N [C_L(A)]$.

The family of all b-open set in the topological space is denoted by F[b-o(X)].

2.2. Proposition : The set of all elements in the real space R with the usual topology T is always a b-open set in the topological space.

We shall show it by taking an example in the real space with the usual topology.

Let X = R ; (the real space) then $C_L(X) = C_L(R) = R$ & $I_N(X) = I_N (R) = R$

Also, $I_N \{C_L (X)\} = I_N \{R\} = R$ & $C_L\{ I_N(X)\} = C_L (R) = R$.

So, $C_L\{ I_N(X)\} \cup I_N\{C_L(X)] = R \cup R = R$. Also, $R \subseteq R$.

Hence, $X \subset C_L\{ I_N(X)\} \cup I_N\{C_L(X)]$. Therefore, the set of all elements in the real space R with the usual topology T is always a b-open set.

2.3. Proposition : The empty set φ is always a b-open set in the real space with the usual topology.

We shall show it by taking an example in the real space R with the usual topology.

Let X = φ ; (the empty set) then $C_L(X) = C_L(\varphi) = \varphi$ & $I_N(X) = I_N (\varphi) = \varphi$.

Also, $I_N \{C_L (X)\} = I_N \{\varphi\} = \varphi$ & $C_L\{ I_N (X)\} = C_L(\varphi) = \varphi$.

So, $C_L\{ I_N(X)\} \cup I_N\{C_L(X)] = \varphi \cup \varphi = \varphi$. Also, $\varphi \subseteq \varphi$.

Hence, $X \subset C_L\{ I_N(X)\} \cup I_N \{C_L(X)\}]$. Therefore, the empty set φ is always a b-open set in the real space with the usual topology.

2.4. Proposition : Every open set is a b-open set in the real space with the usual topology.

We shall show it by taking an example in the real space R with the usual topology.

Let X =]a , b[; (the open interval) then $C_L (X) = C_L( ]a , b[ ) = [a , b]$ & $I_N(X) = I_N \{]a , b[\} =]a , b[$. Also, $I_N \{C_L(X)\} = I_N \{[a , b]\} =]a , b[$ &

$C_L\{ I_N(X)\} = C_L(\ ]a , b[\ ) = [a , b]$.

So, $C_L\{ I_N(X)\} \cup I_N\{C_L(X)\}] = [a , b] \cup]a , b[= [a , b]$. Also, $]a , b[\subseteq [a , b]$.

Hence, $X \subset C_L\{ I_N(X)\} \cup I_N \{C_L(X)\}]$. Therefore, every open interval is a b-open set in the real space with the usual topology.

As we know that every open interval is an open set in the topological space. So, the set X is an open set in the topological space. **Hence, every open set is a b-open set in the real space with the usual topology.**

2.5. <u>Proposition</u> : Every closed set is also a b-open set in the real space with the usual topology.

We shall show it by taking an example in the real space R with the usual topology.

Let $X = [a , b]$; (the closed interval) then $C_L(X) = C_L([a , b]) = [a , b]$ & $I_N(X) = I_N \{ [a , b] \} =]a , b[$. Also, $I_N \{C_L(X)\} = I_N \{[a , b]\} =]a , b[$ & $C_L\{ I_N(X)\} = C_L(\ ]a , b[\ ) = [a , b]$.

So, $C_L\{ I_N(X)\} \cup I_N\{C_L(X)\}] = [a , b] \cup]a , b[= [a , b]$. Also, $[a , b] \subseteq [a , b]$.

Hence, $X \subset C_L\{ I_N(X)\} \cup I_N \{C_L(X)\}]$.Therefore, every closed interval is also a b-open set in the real space with the usual topology.

As we know that every closed interval is always a closed set in the real space. So, the set X is an open set in the topological space. **Hence, every closed set is also a b-open set in the real space with the usual topology.**

2.6 . <u>Proposition</u> : Every half open interval is always a b-open set in the real space with the usual topology.

We shall show it by taking an example in the real space R with the usual topology.

<u>Case - [1]</u> : Let $X = [a , b [$,

Let $X = [a , b [$; (the half open interval) then $C_L(X) = C_L([a , b[ ) = [a , b]$ & $I_N(X) = I_N \{ [a , b [\} =]a , b[$. Also, $I_N \{C_L(X)\} = I_N \{[a , b]\} =]a , b[$ & $C_L\{ I_N(X)\} = C_L(\ ]a , b[\ ) = [a , b]$.

So, $C_L\{ I_N(X)\}$ U $I_N\{C_L(X)]$ = [a , b] U]a , b[= [a , b]. Also, [a , b[⊆ [a , b].

Hence, X ⊂ $C_L\{ I_N(X)\}$ U I_N $\{C_L(X)\}]$. Therefore, every half open interval X = [a , b [is always a b-open set in the real space.

<u>Case - [2]</u> : Let X =] a , b],

Let X =] a , b] ; (the half open interval) then $C_L(X) = C_L($]a , b]) = [a , b] & $I_N(X) = I_N$ {] a , b] } =]a , b[. Also, I_N $\{C_L(X)\}$ = I_N {[a , b]} =]a , b[& $C_L\{ I_N (X)\} = C_L($]a , b[ ) = [a , b].

So, $C_L\{ I_N(X)\}$ U $I_N\{C_L(X)\}]$ = [a , b] U]a , b[= [a , b]. Also,]a , b] ⊆ [a , b].

Hence, X ⊂ $C_L\{ I_N(X)\}$ U I_N $\{C_L(X)\}]$. Therefore, every half open interval X =] a , b] is always a b-open set in the real space.

Hence, every half open interval is always a b-open set in the real space with the usual topology.

As we know that the intersection of two open set is always an open set in the topological space and also this is true in the case of b-open set in the topological space, i.e.,

2.7. <u>Remark</u> : The intersection of two b-open set is may or may not be a b-open set in the topological space.

We shall show it by taking an example in the real space R with the usual topology.

<u>Case - [1]</u> : In the 1st following example, the intersection of two is b-open set a b-open set in the topological space.

We have discussed earlier that the every open interval is b-open set.

Let F =]a , b[∩]b , c[; where, a < b < c.

So, F =]a , b[∩]b , c[= φ & $C_L(F) = C_L(φ)$ = φ & $I_N(F) = I_N$ (φ) = φ. Also, I_N $\{C_L (F)\} = I_N$ {φ} = φ & $C_L\{ I_N (F)\} = C_L(φ)$ = φ.

So, $C_L\{ I_N(F)\}$ U $I_N\{C_L(F)]$ = φ U φ = φ. Also, φ ⊆ φ.

Hence, $F \subset C_L\{ I_N(F)\} \cup I_N \{C_L(F)\}]$. Therefore, the intersection of two b-open set, i.e., $F =]a , b[\cap]b , c[$ is also a b-open set in the topological space.

<u>Case - [2]</u> : In the 2nd following example, in which the intersection of two b-open set is not a b-open set in the topological space.

We have discussed earlier that the every closed interval is the b-open set in the topological space.

Let $F = [a , b] \cap [b , c]$; where, $a < b < c$.

So, $F = [a , b] \cap [b , c] = \{b\}$ & $C_L(F) = C_L(\{b\}) = \varphi$ & $I_N(F) = I_N (\{b\}) = \varphi$.

Also, $I_N \{C_L (F)\} = I_N \{\varphi\} = \varphi$ & $C_L\{ I_N (F)\} = C_L(\varphi) = \varphi$.

So, $C_L\{ I_N(F)\} \cup I_N\{C_L(F)\}] = \varphi \cup \varphi = \varphi$. But, $\{b\} \not\subset \varphi$.

Hence, $F \not\subset C_L\{ I_N(F)\} \cup I_N \{C_L(F)]$. Therefore, the intersection of two b-open set, i.e., $F = [a , b] \cap [b , c]$, is not a b-open set in the topological space.

Hence, the intersection of two b-open set is may or may not be a b-open set in the topological space.

As we know that the union of two open set is always an open set in the topological space and also this is true in the case of b-open set in the topological space, i.e.,

2.8. <u>Proposition</u> : The union of two b-open set is always a b-open set in the topological space.

We shall show it by taking an example in the real space R with the usual topology.

<u>Case - [1]</u> : Let $F =]a , b[\cup]b , c[$, where $a < b < c$.

We have discussed earlier that the every open interval is the b-open set in the topological space.

Let $F =]a , b[\cup]b , c[$, $a < b < c$.

So, $F =] a , b [\cup] b , c [=] a , c [- \{b\}$ then $C_L (F) = C_L( ] a , c [- \{b\}) = [a , c]$ & $I_N(F) = I_N ( ] a , c [- \{b\}) =]a , c[$. Also, $I_N\{C_L(F)\} = I_N \{[a , c]\} =]a , c[$ & $C_L\{I_N(F)\} = C_L[\{] a , c [\}] = [a , c]$.

So, $C_L\{ I_N(F)\} \cup I_N\{C_L(F)] = [a , c] \cup]a , c[= [a , c]$.

Also, $]a , c[- \{b\} \subseteq [a , c]$. **Hence, $F \subset C_L\{ I_N(F)\} \cup I_N \{C_L(F)]$.**

Therefore, the union of two b-open set, i.e., F =]a , b[U]b , c[is a b-open set in the topological space.

<u>Case - [2]</u> : Let F = [a , b] U [b , c], a < b < c .

We have discussed earlier that the every closed interval is the b-open set in the topological space.

Let $F = [a , b] \cup [b , c]$; where, $a < b < c$.

So, $F = [a , b] \cup [b , c] = [a , c]$ then $C_L(F) = C_L([a , c]) = [a , c]$ &

$I_N(F) = I_N \{ [a , c] \} =]a , c[$. Also, $I_N \{C_L(F)\} = I_N \{[a , c]\} =]a , c[$ &

$C_L\{ I_N (F)\} = C_L( ]a , c[ ) = [a , c]$.

So, $C_L\{ I_N(F)\} \cup I_N\{C_L(F)] = [a , c] \cup]a , c[= [a , c]$. Also, $[a , c] \subseteq [a , c]$.

Hence, $F \subset C_L\{ I_N(F)\} \cup I_N \{C_L(F)]$.

Therefore, the union of two b-open set, i.e., F = [a , b] U [b , c] is a b-open set in the topological space.

Hence, in any condition the union of two b-open set is always a b-open set in the topological space.

Next, we have to show that this is also true in the case of arbitrary collection of b-open set, i.e.,

2.9. <u>Proposition</u> : The union of arbitrary collection of b-open set is also a b-open set in the topological space.

<u>Verification</u> :

Let us consider, $F_n = [\frac{1}{n} , 2] \quad \forall n \in N$.

Then $F_1 = [1 , 2]$, $F_2 = [\frac{1}{2} , 2]$, $F_3 = [\frac{1}{3} , 2]$, $F_4 = [\frac{1}{4} , 2]$ ……..

Since, every closed interval is a b-open set as above .

Therefore, $F_n = [\frac{1}{n}, 2] \ \forall n \in N$ is a family of infinite number of b-open set.

Now, $F_1 \cup F_2 = [1, 2] \cup [\frac{1}{2}, 2] = [\frac{1}{2}, 2]$,

$F_1 \cup F_2 \cup F_3 = [1, 2] \cup [\frac{1}{2}, 2] \cup [\frac{1}{3}, 2] = [\frac{1}{3}, 2]$,

$F_1 \cup F_2 \cup F_3 \cup F_4 = [1, 2] \cup [\frac{1}{2}, 2] \cup [\frac{1}{3}, 2] \cup [\frac{1}{4}, 2] = [\frac{1}{4}, 2]$ and so on.

Indeed, $\bigcup_{n=1}^{\infty}(F_n) =]0, 2]$ and $]0, 2]$ is also a b-open set as above.

Hence, the union of arbitrary collection of b-open set is always a b-open set in the topological space.

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