Group action of some Topological spaces Via supra B -open and supra β- open sets

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Abstract:

In this paper, we introduce and investigate a new class of sets and maps between topological spaces called supra B -open and supra β-open, respectively. Furthermore, we introduce the concepts of group action of supra B -open and supra B -open map, supra B -continuous and supra β-open.

Keywords: Group acting on supra b-open set, supra B -continuous, supra b-open map, supra β-open set and using topological spaces.

1. Introduction:

In 1996, D.Andrijivic introduced and studied a class of generalized open sets in a topological spaces called B -open sets. This class of sets contained in the class of β-open sets. In 1983, A.S.Mashhour et al. Introduced the supra topological spaces and studied s-continuous map and s*-continuous maps. In 2010, O.R.sayed and Takashi Noiri Introduced and studied a class of sets and map between topological spaces called supra B -open and supra B -continuous respectively. Now we introduced the concepts of group action of supra B -open, supra B -open map, supra B -continuous and supra β-open sets.

2. Preliminaries:

Definition: 2.1

A collection μ of subsets of 2^x is said to be a supra topology on X if X belongs to μ and the union of an arbitrary family of sets in μ belongs to μ. A pair (X,μ) is called a supra topological space. Every member of μ is said to be supra open and its complement is said to be supra closed.

Definition: 2.2

Let A be a subset of (X,μ). Then the supra closure of A, denoted by $cl^{\mu}(A)$, is the intersection of all supra closed sets containing A and the supra interior of A, denoted by $Int^{\mu}(A)$, is the union of all supra open sets contained in A.

Definition:2.3

A subset A of (X,μ) is called supra semi-open if $A \subseteq cl^{\mu}(Int^{\mu}(A))$. The complement of a supra semi-open set is called supra semi-closed.

Definition: 2.4

Let G be a topological group this means that G is a topological space and also a group so that the multiplication map $\mu: G \times G \to G$.

$\mu(g.h) = gh$ and the inversemap $i: G \to G$

$i(g) = g^{-1}$ are continuous.

Definition: 2.5

Let (X,μ) be a supra topological space. A set A is called a supra B -open set if $A \subseteq cl^{\mu}(Int^{\mu}(A)) \cup Int^{\mu}(cl^{\mu}(A))$. The complement of a supra B -open set is called a supra B -closed set.

3. Supra B -open set

Theorem 3.1

Group action of supra semi open set is supra B -open.

Proof:

Let A_G be a supra set in topological group (X,μ) then $A_G \subseteq cl^{\mu}(Int^{\mu}(A_G))$. Hence $A_G \subseteq cl^{\mu}(Int^{\mu}(A_G)) \cup Int^{\mu}(cl^{\mu}(A_G))$ and $A_G$is supra B -open in (X,μ).

Theorem 3.2

G acted on arbitrary union of supra B -open sets in always supra B -open set.

Proof:

Let $A_G \subseteq cl^{\mu}(Int^{\mu}(A_G)) \cup Int^{\mu}(cl^{\mu}(A_G))$ and

$B_G \subseteq cl^{\mu}(Int^{\mu}(B_G)) \cup Int^{\mu}(cl^{\mu}(B_G))$ then

$A_G \cup B_G \subseteq cl^{\mu}(Int^{\mu}(A_G \cup B_G)) \cup Int^{\mu}(cl^{\mu}(A_G \cup B_G))$

Therefore $A_G \cup B_G$ is G acted on supra B -open set.

Supra B –continuous map:

Let (X,τ_1) and (Y,τ_2) be two topological spaces and μ be an associated supra topology with τ_1. A map $f: (X,\tau_1) \to (Y,\tau_2)$ is called a supra B -continuous map if the inverse image of each open set in Y is called supra B -open map set in X.

Theorem 3.3

Group action of every continuous map is group action of B -continuous.

Proof:

Let $g: (X,\tau_1) \to (Y,\tau_2)$ be a continuous map and A_G is open in Y.
Then $g^{-1}(A_G)$ is an open set in X. since μ is associated with $\tau_1$then $\tau_1 \subseteq \mu$

Therefore $g^{-1}(A_G)$ is supra open in X and it is supra b-open in X
Hence g is group acted on supra B –continuous.

Theorem 3.4

Let (X, τ_1) and (Y, τ_2) be two topological group and μ be an associated supra topology with τ_1. Let g be a map from X into Y. Then the following equivalent

(i) g is a group action of supra B -continuous map.
(ii) The inverse image of a closed set in Y is a group action of supra B -closed set in X.
(iii) ${cl_b}^{\mu}(g^{-1}(A_G)) \subseteq g^{-1}(cl(A_G))$ for every set A_G in Y.
(iv) $g({cl_b}^{\mu}(A_G)) \subseteq cl(g(A_G))$ for every set A_G in X.
(v) $g^{-1}(Int(B_G)) \subseteq {Int_b}^{\mu}(g^{-1}(B_G))$ for every B_G in Y.

Proof:

$(i) \Longrightarrow (ii)$

Let A_G be a closed set in Y then $Y - A_G$ is an open set in Y. Then $g^{-1}(Y - A_G) = X - g^{-1}(A_G)$ is a group action of supra B -open set in X.
It follows that $g^{-1}(A_G)$ is a G acted on supra B -closed set of X.

$(ii) \Longrightarrow (iii)$

Let A_G be any subset of Y. since $cl(A_G)$ is closed in Y,
then $g^{-1}(cl(A_G))$ is g acted on supra B -closed set of X.
${cl_b}^{\mu}\left(g^{-1}(A_G)\right) \subseteq {cl_b}^{\mu}\left(g^{-1}\left(cl(A_G)\right)\right) = g^{-1}(cl(A_G))$.

$(iii) \Longrightarrow (iv)$

Let A_G be any subset of X, by (iii) we have
$g^{-1}(cl\left(g(A_G)\right)) \supseteq {cl_b}^{\mu}(g^{-1}\left(g(A_G)\right)) \supseteq {cl_b}^{\mu}(A_G)$.
Therefore $g({cl_b}^{\mu}(A_G)) \subseteq cl(g(A_G))$.

$(iv) \Longrightarrow (v)$

Let B_G be any subset of Y
by (iv) we have $g({cl_b}^{\mu}\left(X - g^{-1}(B_G)\right)) \subset cl(g\left(X - g^{-1}(B_G)\right))$ and

$$g(X - {Int_b}^{\mu}\left(g^{-1}(B_G)\right)) \subset cl(Y - B_G) = Y - Int(B_G)$$

Therefore we have $X - {Int_b}^{\mu}(g^{-1}(B_G)) \subset g^{-1}(Y - Int(B_G))$ and

$$g^{-1}(Int(B_G)) \subset {Int_b}^{\mu}(g^{-1}(B_G))$$

$(v) \Longrightarrow (i)$

Let B_G be an open set in Y and $g^{-1}(Int(B_G)) \subseteq {Int_b}^{\mu}(g^{-1}(B_G))$
Then $g^{-1}(B_G) \subseteq {Int_b}^{\mu}(g^{-1}(B_G))$ but ${Int_b}^{\mu}\left(g^{-1}(B_G)\right) \subseteq g^{-1}(B_G)$
Hence $g^{-1}(B_G) = {Int_b}^{\mu}(g^{-1}(B_G))$
Therefore, $g^{-1}(B_G)$ is group action of supra B -open in X.

Supra B – open map:

A map $g: (X, \tau_1) \to (Y, \tau_2)$ is called a supra B -open (resp. supra B -closed) if the image of each open (resp. closed) set in X is supra B -open (resp. supra B closed) in (Y, v).

Theorem 3.5

A map $g: (X, \tau_1) \to (Y, \tau_2)$ is Group acted on supra B -open if and only if $g(Int\,(A_G)) \subseteq {Int_b}^v(g(A_G))$ for each set A_G in X.

Proof:

Suppose that g is a Group acted on supra B – open map
since $Int(A_G) \subseteq A_G$, then $g(Int(A_G)) \subseteq g(A_G)$
$g(Int(A_G))$ is a G acted on supra B -open set and ${Int_b}^v(g(A_G))$ is the g acted on largest supra b-open set contained in $g(A_G)$.
Hence $g(Int(A_G)) \subseteq {Int_b}^v(g(A_G))$.

Conversely, suppose A_G is an open set in X. Then $g(Int(A_G)) \subseteq {Int_b}^v(g(A_G))$ since $Int(A_G) = A_G$, then $g(A_G) \subseteq {Int_b}^v(g(A_G))$.
Therefore $g(A_G)$ is a group action of supra B -open set in (Y, v) and g is a group action of supra B -open map.

Theorem : 3.6

Let (X, τ_1) and (Y, τ_2) be two topological group and $g: (X, \tau_1) \to (Y, \tau_2)$ be a bijective map. Then the following are equivalent:

(i) g is a group action of supra B -open map.
(ii) g is a group action of supra B -closed map.
(iii) $g^{-1}$is a supra B -continuous map.

Proof:

$(i) \Rightarrow (ii)$

suppose B_G is a closed set in X. then $X - B_G$ is an open set in X and
by (i) $g(X - B_G)$ g acted on B – open set in Y.
since g is bijective, then $g(X - B_G) = Y - g(B_G)$.
Hence $g(B_G)$ is a g acted on supra B –closed set in Y.
Therefore g is a g acted on supra B -closed map.

$(ii) \Rightarrow (iii)$

Let g is a g acted on supra B -closed map and B_G be closed set in X.
since g is bijective then $(g^{-1})^{-1}(B_G) = g(B_G)$ which is a g acted on supra B -closed set in Y.
Therefore, by Theorem 1, g is a group action of supra B -continuous map.

$(iii) \Rightarrow (i)$

Let A_G be an open set in X.

since g^{-1} is a group acted on supra B -continuous map, then $(g^{-1})^{-1}(A_G) = g(A_G)$ is g acted on supra B –open set in Y.

Hence, g is a group action of supra B –open map.

4. Supra β open set:

Let (X, μ) be a supra topological space. A set A is called a supra β -open set if $A \subseteq cl^{\mu}(Int^{\mu}(cl^{\mu}(A)))$. The complement of a supra β -open set is called a supra β -closed set.

Theorem: 4.1

Group action of supra semi open set is supra β -open.

Proof:

Let A_G be a supra set in topological group (X, μ) then $A_G \subseteq cl^{\mu}(Int^{\mu}(A_G))$.

Hence $A_G \subseteq cl^{\mu}(Int^{\mu}(cl^{\mu}(A_G)))$ and $A_G$is supra β -open in (X, μ).

Theorem 4.2

G acted on arbitrary union of supra β -open sets in always supra β -open set.

Proof:

Let $A_G \subseteq cl^{\mu}(Int^{\mu}(cl^{\mu}(A_G)))$ and

$B_G \subseteq cl^{\mu}(Int^{\mu}(cl^{\mu}(B_G)))$ then

$A_G \cup B_G \subseteq cl^{\mu}(Int^{\mu}(cl^{\mu}(A_G \cup B_G)))$

Therefore $A_G \cup B_G$ is G acted on supra β -open set.

Supra β –continuous map:

Let (X, τ_1) and (Y, τ_2) be two topological spaces and μ be an associated supra topology with τ_1. A map $f: (X, \tau_1) \to (Y, \tau_2)$ is called a supra β -continuous map if the inverse image of each open set in Y is called supra β - open map set in X.

Theorem 4.3

Group action of every continuous map is group action of β -continuous.

Proof:

Let $g: (X, \tau_1) \to (Y, \tau_2)$ be a continuous map and A_G is open in Y.

Then $g^{-1}(A_G)$ is an open set in X. since μ is associated with $\tau_1$then $\tau_1 \subseteq \mu$ therefore $g^{-1}(A_G)$ is supra open in X and it is supra β -open in X

Hence g is group acted on supra β – continuous.

Theorem 4.4

Let (X, τ_1) and (Y, τ_2) be two topological group and μ be an associated supra topology with τ_1. Let g be a map from X into Y. Then the following equivalent

(i) g is a group action of supra β -continuous map.

(ii) The inverse image of a closed set in Y is a group action of supra β –closed set in X.

(iii) ${cl_\beta}^\mu(g^{-1}(A_G)) \subseteq g^{-1}(cl(A_G))$ for every set A_G in Y.

(iv) $g({cl_\beta}^\mu(A_G)) \subseteq cl(g(A_G))$ for every set A_G in X.

(v) $g^{-1}(Int(B_G)) \subseteq {Int_\beta}^\mu(g^{-1}(B_G))$ for every B_G in Y.

Proof:

$(i) \Longrightarrow (ii)$

Let A_G be a closed set in Y, then $Y - A_G$ is an open set in Y. Then $g^{-1}(Y - A_G) = X - g^{-1}(A_G)$ is a group action of supra β -open set in X. It follows that $g^{-1}(A_G)$ is a G acted on supra β -closed set of X.

$(ii) \Longrightarrow (iii)$

Let A_G be any subset of Y. since $cl(A_G)$ is closed in Y, then $g^{-1}(cl(A_G))$ is g acted on supra β -closed set of X. Therefore ${cl_\beta}^\mu\left(g^{-1}(A_G)\right) \subseteq {cl_\beta}^\mu\left(g^{-1}\left(cl(A_G)\right)\right) = g^{-1}(cl(A_G))$

$(iii) \Longrightarrow (iv)$

Let A_G be any subset of X, by (iii) we have
$g^{-1}(cl\left(g(A_G)\right)) \supseteq {cl_\beta}^\mu(g^{-1}\left(g(A_G)\right)) \supseteq {cl_\beta}^\mu(A_G)$.
Therefore $g({cl_\beta}^\mu(A_G)) \subseteq cl(g(A_G))$.

$(iv) \Longrightarrow (v)$

Let B_G be any subset of Y
by (iv) we have $g({cl_\beta}^\mu\left(X - g^{-1}(B_G)\right)) \subset cl(g\left(X - g^{-1}(B_G)\right))$ and

$$g(X - {Int_\beta}^\mu\left(g^{-1}(B_G)\right)) \subset cl(Y - B_G) = Y - Int(B_G)$$

Therefore we have $X - {Int_\beta}^\mu(g^{-1}(B_G)) \subset g^{-1}(Y - Int(B_G))$ and
$g^{-1}(Int(B_G)) \subset {Int_\beta}^\mu(g^{-1}(B_G))$.

$(v) \Longrightarrow (i)$

Let B_G be an open set in Y and $g^{-1}(Int(B_G)) \subseteq {Int_\beta}^\mu(g^{-1}(B_G))$
Then $g^{-1}(B_G) \subseteq {Int_\beta}^\mu(g^{-1}(B_G))$ but ${Int_\beta}^\mu\left(g^{-1}(B_G)\right) \subseteq g^{-1}(B_G)$
Hence $g^{-1}(B_G) = {Int_\beta}^\mu(g^{-1}(B_G))$
Therefore, $g^{-1}(B_G)$ is group action of supra β -open in X.

Supra β – open map:

A map $g:(X,\tau_1)\rightarrow(Y,\tau_2)$ is called a supra β -open (resp. supra b-closed) if the image of each open (resp. closed) set in X is supra β -open (resp. supra b closed) in (Y,v).

Theorem 4.5

A map $g:(X,\tau_1)\rightarrow(Y,\tau_2)$ is Group acted on supra β -open if and only if $g(Int\ (E_G))\subseteq Int_\beta{}^v(g(E_G))$ for each set A_G in X.

Proof:

Suppose that g is a Group acted on supra β – open map
since $Int(E_G)\subseteq E_G$, then $g(Int(E_G))\subseteq g(E_G)$
$g(Int(E_G))$ is a G acted on supra β -open set and $Int_\beta{}^v(g(E_G))$ is the g acted on largest supra β -open set contained in $g(E_G)$.
Hence $g(Int(E_G))\subseteq Int_\beta{}^v(g(E_G))$.

Conversely, suppose A_G is an open set in X. Then $g(Int(E_G))\subseteq Int_\beta{}^v(g(E_G))$
since $Int(E_G)=E_G$, then $g(E_G)\subseteq Int_\beta{}^v(g(E_G))$.
Therefore $g(E_G)$ is a group action of supra β -open set in (Y,v) and g is a group action of supra β -open map.

Theorem : 4.6

Let (X,τ_1) and (Y,τ_2) be two topological group and $g:(X,\tau_1)\rightarrow(Y,\tau_2)$ be a bijective map. Then the following are equivalent:

(i) g is a group action of supra β -open map.
(ii) g is a group action of supra β -closed map.
(iii) $g^{-1}$is a supra β -continuous map.

Proof:

$(i)\Rightarrow(ii)$

suppose E_G is a closed set in X. then $X-E_G$ is an open set in X and
by (i) $g(X-E_G)$ g acted on β – open set in Y.
since g is bijective, then $g(X-E_G)=Y-g(E_G)$.
Hence $g(E_G)$ is a g acted on supra β –closed set in Y.
Therefore g is a g acted on supra β -closed map.

$(ii)\Rightarrow(iii)$

Let g is a g acted on supra b-closed map and B_G be closed set in X.
since g is bijective then $(g^{-1})^{-1}(E_G)=g(E_G)$ which is a g acted on supra β -closed set in Y.
Therefore, by Theorem 1, g is a group action of supra β -continuous map.
$(iii)\Rightarrow(i)$

Let E_G be an open set in X.
since g^{-1} is a group acted on supra β -continuous map, then $(g^{-1})^{-1}(E_G) = g(E_G)$ is g acted on supra β –open set in Y.
Hence, g is a group action of supra β –open map.

Conclusion:

In this paper we introduce the basic concepts of supra b- open set, supra b-continuous, supra b-open map, supra β open set, supra β -continuous, supra β -open map of topological spaces. More over we introduce the concepts of group actions of related to supra b –open and supra β –open sets of topological spaces. Further study several topics in group actions of topological spaces.

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